The University of Hong Kong
MS 19-601-15/93: Finance

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Assignment #1 Solutions

1. (12 points) The following table shows the yields of U.S. Treasuries on April 1, 2010

Maturity	1 yr	2 yr	3 yr	5 yr	7 yr
Yield	0.42	1.05	1.63	2.59	3.32

Assume that the Pure Expectations Theory holds.

a. What is the expected yield of a 1-year Treasury <u>two</u> years from now?

$$\begin{aligned} r_3 &= \frac{2r_2 + ({}_2r_1^e)}{3} \\ 1.63 &= \frac{2(1.05) + ({}_2r_1^e)}{3} \\ {}_2r_1^e &= 3(1.63\%) - 2(1.05\%) \\ &= 2.79\% \end{aligned}$$

b. What is the expected yield of a 2-year Treasury <u>three</u> years from now?

$$\begin{aligned} r_5 &= \frac{3(r_3) + 2({}_3r_2^e)}{5} \\ 2.59 &= \frac{3(1.63) + 2({}_3r_2^e)}{5} \\ {}_3r_2^e &= \frac{12.95 - 4.89}{2} = 4.03\% \end{aligned}$$

Suppose the term premium on a three year bond is estimated to be 0.25%.

c. What is the expected yield of a 1-year Treasury <u>two</u> years from now according to the liquidity premium theory?

$$\begin{aligned} r_3 &= \frac{2r_2 + ({}_2r_1^e)}{3} + \ell_3 \\ 1.63 &= \frac{2(1.05) + ({}_2r_1^e)}{3} + 0.25 \\ {}_2r_1^e &= 3(1.38\%) - 2(1.05\%) \\ &= 2.04\% \end{aligned}$$

2. (10 points) Two stores have the same list price of $12,000 for a HDTV you are interested in. Store A offers a payment plan of $2,000 down payment and the balance to be paid in two equal annual installments beginning next year with no finance charge. Store B offers a plan with no money down and three equal annual payments of $4,000 beginning next years, also with no finance charge. Assume that the market interest rate is 4%. Which store offers a better deal? Show all your work and explain.

The PV of these two options are

$$\begin{aligned} \text{PV}_A &= 2,000 + \frac{5,000}{(1+0.04)} + \frac{5,000}{(1+0.04)^2} \\ &= 2,000 + 4,807.69 + 4,622.47 = \$11,430.47 \end{aligned}$$

$$\begin{aligned} \text{PV}_B &= \frac{4,000}{(1+0.04)} + \frac{4,000}{(1+0.04)^2} + \frac{4,000}{(1+0.04)^3} \\ &= 3,846.15 + 3,698.22 + 3,555.99 = \$11,100.36 \end{aligned}$$

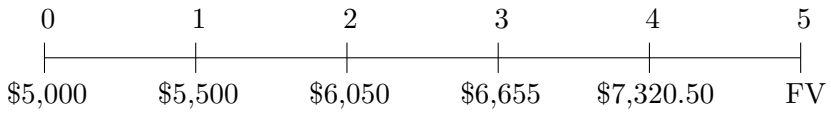
Store B offers a better deal because $\text{PV}_A > \text{PV}_B$.

3. (24 points) You open an investment account with \$5,000 today ($t = 0$) to save for the down payment on a house. Your will make four additional deposits on the same day in the next four years. Following your brother's suggestion, you will increase the amount of deposit by 10 percent each year.

(8) a. Draw a clear and accurate Time Line of your financial plan.

First work out the payment of each period: $\text{PMT}_t = \$5,000(1.10)^t, t = 0, 1, \ldots 4$.

$\text{PMT}_0 = \$5,000; \text{PMT}_1 = \$5,500; \text{PMT}_2 = \$6,050;$
$\text{PMT}_3 = \$6,655; \text{PMT}_4 = \$7,320.50.$



(8) b. Suppose the account pays 12% interest per year, compounded quarterly. How much money will you have in your account at the end of the fifth year?

Ans: \$43,151.94

$$\begin{aligned} FV &= \$5,000(1.03)^{20} + \$5,500(1.03)^{16} + \$6,050(1.03)^{12} + \$6,655(1.04)^8 + \\ &\qquad \$7,320.50(1.03)^4 \\ &= 9,030.56 + 8,825.89 + 8,625.85 + 8,430.35 + 8,239.29 \\ &= \$43,151.94. \end{aligned}$$

Once you saved enough money for the down payment, you plan to take out a 10-year mortgage of \$400,000 from a bank to finance the house payments.

(8) c. If the mortgage rate is 6%, how much will your monthly mortgage payment be?

Ans:

$$\begin{aligned} \text{PMT} &= \text{PV}_A \frac{i}{1-(1+i)^{-n}} \\ &= 400,000 \frac{0.06/12}{1-(1+0.06/12)^{-120}} \\ &= 4,440.82 \end{aligned}$$

4. (10 points) Consider a bond which has 10 years until maturity, a coupon rate of 6.0%, makes semi-annual payments, and a YTM of 7.5%.

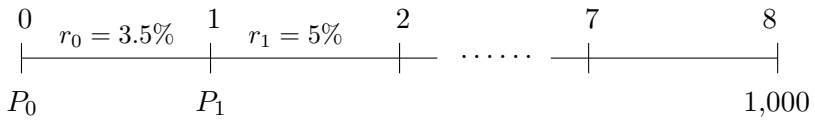
   a. Show that the price of this bond is \$895.778.

$$\begin{aligned} P &= 30\left(\frac{1}{\frac{0.075}{2}} - \frac{1}{\frac{0.075}{2}(1+\frac{0.075}{2})^{20}}\right) + \frac{1,000}{(1+\frac{0.075}{2})^{20}} \\ &= 30(26.6667 - 12.7705) + 478.892 \\ &= \$895.778 \end{aligned}$$

   b. Suppose you bought this bond and sold it a year later for \$928.938. What is your rate of return on this bond?

$$\text{Rate of Return} = \frac{928.938 - 895.778 + 60}{895.778} = \frac{93.16}{895.778} = 10.395\%$$

5. (8 points) Consider an 8-year **zero-coupon bond** with a \$1,000 par value and 3.5% yield to maturity. If interest rate rises to 5% a year later, will the price of this bond increase or decrease? By how much?



$$\begin{aligned} \text{(Original Price) } P_0 &= \frac{\$1,000}{(1+0.035)^8} = 759.412 \\ \text{(Price one year later) } P_1 &= \frac{\$1,000}{(1+0.05)^7} = 710.681 \\ \text{Price decrease: } P_1 - P_0 &= \$710.681 - \$759.412 \\ &= -\$48.73 \end{aligned}$$

6. (16 points) Consider a portfolio of two stocks, A and B. The rate of return on the two stocks depends on three possible states of the economy. The probability of each state occurring and rates of return for Stocks A and B associated with each state are estimated as follows:

Economy	Prob.	A	B
Boom	0.1	15.6%	25.3%
Normal	0.6	10.5%	6.4%
Recession	0.3	6.3%	−3.7%

(4) a. Find the expected return on each stock.

$$\begin{aligned}\mu_A &= \frac{1}{10}(0.156)+\frac{6}{10}(0.105)+\frac{3}{10}(0.063)=0.09751=9.75\%\\ \mu_B &= \frac{1}{10}(0.253)+\frac{6}{10}(0.064)+\frac{3}{10}(-0.037)=0.0526=5.26\%\end{aligned}$$

(4) b. Find the standard deviation of return on each stock.

$$\begin{aligned}\sigma_A &= \sqrt{\frac{1}{10}(0.156-0.0975)^2+\frac{6}{10}(0.105-0.0975)^2+\frac{3}{10}(0.063-0.0975)^2}\\ &= \sqrt{0.000342+0.000034+0.000357}\\ &= \sqrt{0.000733}\\ &= 0.02707.\\ \sigma_B &= \sqrt{\frac{1}{10}(0.253-0.0526)^2+\frac{6}{10}(0.064-0.0526)^2+\frac{3}{10}(-0.037-0.0526)^2}\\ &= \sqrt{0.004016+0.00007798+0.002408}\\ &= \sqrt{0.006502}\\ &= 0.08064.\end{aligned}$$

(8) c. Find the covariance and the correlation coefficient of returns between the two stocks.

$$\begin{aligned}\sigma_{AB} &= \frac{1}{10}(0.156-0.0975)(0.253-0.0526)\\ &\quad+\frac{6}{10}(0.105-0.0975)(0.064-0.0526)\\ &\quad+\frac{3}{10}(0.063-0.0975)(-0.037-0.0526)\\ &= 0.001172+0.000051+0.000927\\ &= 0.002151.\end{aligned}$$

$$\begin{aligned}\rho_{AB} &= \frac{\sigma_{AB}}{\sigma_A\sigma_B}\\ &= \frac{0.002151}{(0.02707)(0.08064)}=0.9852\end{aligned}$$