Lecture 6
CAPM & Asset Pricing

- Theory of Portfolio Choice: Review
- Capital Asset Pricing Model (CAPM):
  - The Market for Risk
    - Capital Market Line (CML)
  - Pricing Risky Assets
    - Security Market Line (SML)
- Alternative Models of Asset Pricing
  - Arbitrage Pricing Theory (APT)
  - French-Fama 3-Factor Model

Theory of Portfolio Choice & Capital Asset Pricing

Modern Portfolio Theory explores two main issues in finance:

- How do investors make investment choices to maximize returns, given their risk preferences?
  - i.e., How to find an optimum investment portfolio?
- How are prices of capital assets determined in an efficient market?
  - i.e., How do capital markets price risk?

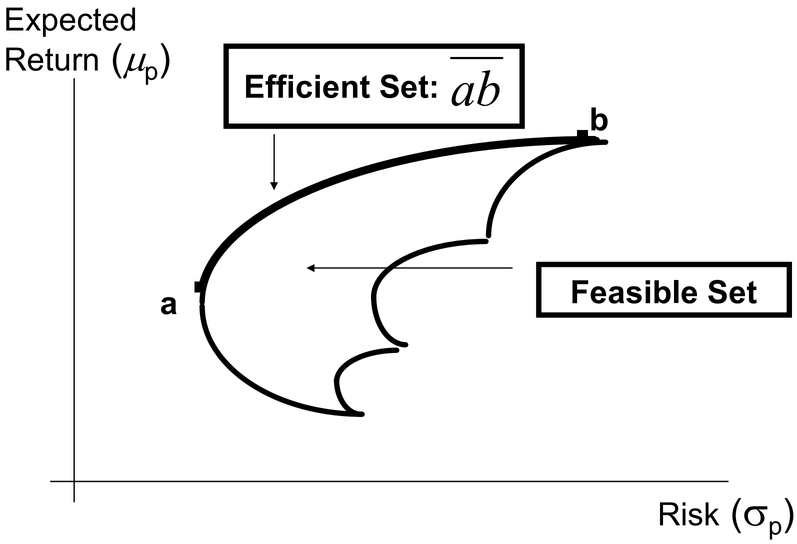
Theory of Portfolio Choice

- The Theory of Portfolio Choice is developed by Harry Markowiz in the 50's.
- The key insight of Markowiz is that the relevant information about securities can be summarized by three statistical measures:
  1. The mean returns (μ)
  2. The standard deviation of returns (σ)
  3. The correlation of returns with that of other securities (ρ)

Theory of Portfolio Choice

- For any given set of risky assets – their (μ_i, σ_i) and ρ_{ij} – we can construct
  - a feasible set: the set of all feasible portfolios or μ-σ combinations, and
  - an efficient frontier: the set of all portfolio that offers the highest expected return for a given amount of risk or the least amount of risk for a given expected return.

Feasible and Efficient Portfolios



Rationality and Risk Aversion

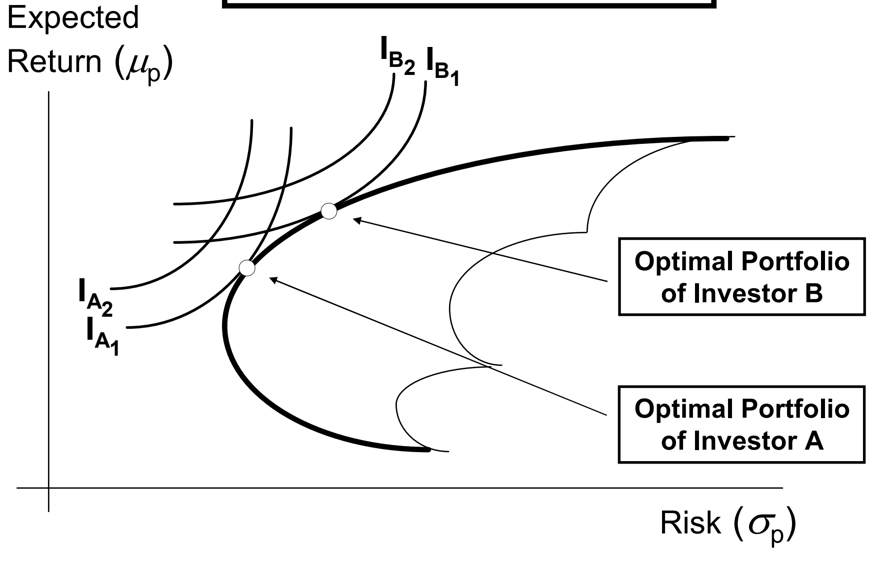
Assumptions on investors behavior:

- Investors care about two things when making investment choices: expected return (μ) and risk (σ).
- Investors are rational and risk averse in the sense that when choosing between assets or portfolios with the same expected returns will prefer the one with the lowest risk.
- Indifference curves are upward sloping because μ is a “good” while σ is a “bad”.

Optimal Portfolio

- An investor's objective is to choose a feasible portfolio on the efficient frontier that can put him/her on the highest indifference curve.
- An investor's optimal portfolio is one at which the efficient frontier is tangent to the highest indifference curve.
- In our example, Investor A is more risk-averse and the (μ,σ) of her optimal portfolio is lower than that of Investor B.
- i.e., a more risk-averse investor will choose a portfolio with lower risk and lower return.

Optimal Portfolio



The Capital Asset Pricing Model (CAPM)

- The CAPM is an **equilibrium** model that describes how the prices of individual assets are determined in an efficient market.
- By providing a precise relationship between an asset's risk and its required return, CAPM helps explain:
  - Why different assets have different required returns
  - Why risk premium exists and how it is determined.

Key Assumptions of the CAPM

CAPM extends Markowitz's portfolio choice model by adding two key assumptions:

1. There are unrestricted borrowing or lending at the risk-free rate.
2. Homogeneous expectations: all investors have the same information on the *distribution* of expected asset returns, hence the same estimates on μ, σ, and ρ.

Other Simplifying Assumptions

- All assets are perfectly divisible.
- There are no taxes and no transactions costs.
- All investors are price takers, i.e., investors' buying and selling won't influence stock prices.
- The quantities of all assets are given and fixed.

The Market Portfolio

- The CAPM assumes that all investors will use Markowitz's portfolio choice model to locate their optimal portfolio on the efficient frontier.
- Suppose we can construct portfolios that include ALL traded assets in the market – think of it as a giant mutual funds.
- We will get a feasible set and an efficient frontier as before.

The Risk-Free Asset

- Next we introduce a risk-free asset like T-Bills (short-term government bonds) which pay a risk-free rate r_{RF}.
- The risk-free rate has no variance in return (i.e. σ_{RF} = 0) and is therefore uncorrelated with the return of other assets (i.e. $\rho_{\mathrm{RF},i}$ = 0 for any asset i).
- Therefore $(\mu_{\mathrm{RF}},\sigma_{\mathrm{RF}}) = (r_{\mathrm{RF}},0)$

The Efficient Frontier with the Risk-Free Asset

- Investors can now form portfolios that combine the risk-free asset with a portfolio of all risky assets (let’s call it M).
- How would that affect the efficient frontier?
- You might think “nothing!" since a risk-free asset provides no diversification *per se*.
- Let's find the μ and σ of all portfolios that can be created and derive the “new” efficient frontier.

(μ_p,σ_p) of Portfolios with a Risk-Free Asset

- Expected Portfolio Return

$$\mu_p = w_{RF} r_{RF} + w_M \mu_M$$

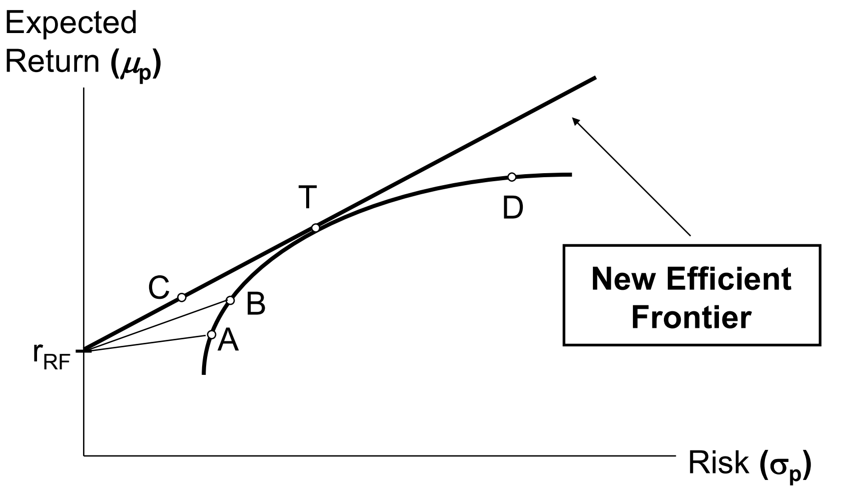
- Portfolio Standard Deviation

$$\sigma_p = \sqrt{w_{RF}^2 \sigma_{RF}^2 + w_M^2 \sigma_M^2 + 2\rho_{RF,M} w_{RF} \sigma_{RF} w_M \sigma_M}$$
$$= \sqrt{w_{RF}^2 (0) + w_M^2 \sigma_M^2 + 2(0) w_{RF} \sigma_{RF} w_M \sigma_M}$$
$$= \sqrt{w_M^2 \sigma_M^2}$$
$$\sigma_p = w_M \sigma_M$$

Efficient Frontier with a Risk-Free Asset

- When we combine a risk-free asset with a portfolio of risky assets, the new (μ_p, σ_p) both vary linearly with their relative weights!
- There is only one optimal risky portfolio: the tangency portfolio T.
- The new efficient frontier will be a straight line extending from r_{RF} through T.
- The efficient frontier extends beyond T because we allow borrowing at the risk-free rate.

Feasible Portfolios & The Efficient Frontier
The Risk-Free Asset + Risky Portfolios



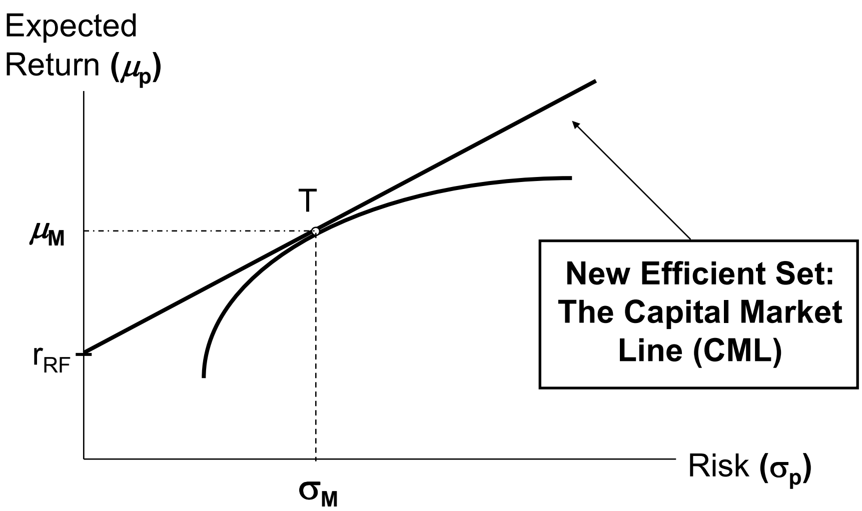
Market Portfolio & Efficiency Frontier

- Since all investors have the same estimates on μ, σ, and ρ (the homogeneous expectations assumption), they will all "see" the same efficient frontier.
- With only one risk-free asset, there is only one efficient portfolio of risky assets: the tangency portfolio T.
- *In equilibrium, all investors will hold the same optimal mix of risky assets. So the tangency portfolio must equal to the market portfolio.*

Market Portfolio & Efficiency Frontier

- In other words, the tangency portfolio T is the market portfolio with (μ_M, σ_M)
- Technically, the market portfolio consists of every traded asset weighted by its market value relative to the entire market.
- The "new" efficient frontier is called the Capital Market Line.

The Capital Market Line (CML)



The Capital Market Line (CML)

- The Capital Market Line (CML) is given by the equation:

$$\mu_p = r_{RF} + (\frac{\mu_M - r_{RF}}{\sigma_M}) \cdot \sigma_p$$

Vertical Intercept (r_{RF}) — Slope — Risk measure (σ_p)

What does the CML tell us?

- The expected rate of return on any efficient portfolio (those on the CML) is equal to the risk-free rate (r_{RF}) plus a risk premium.
- The CML, therefore, gives us **the risk-return relationship of efficient portfolios**.

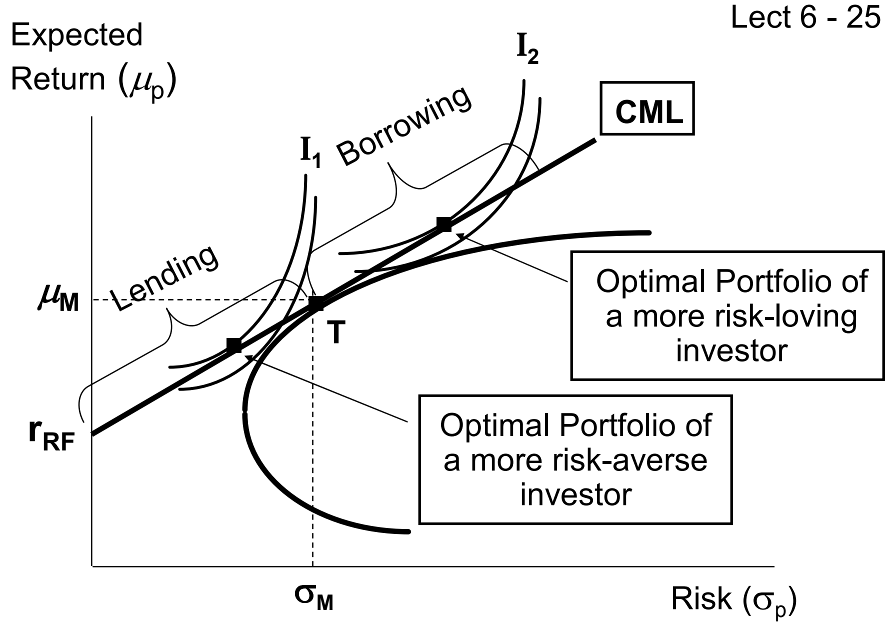
* FYI: The "slope" of the CML is called the Sharpe Ratio (named after William F. Sharpe who was awarded the Nobel Prize in 1990 for creating CAPM), a risk-adjusted measure of return.

Optimal Portfolio with A Risk-Free Asset

- All investors will invest in the market portfolio T on the CML.
- What combination of "risky" and "risk-free" assets is optimal depends on the investor's risk preferences.
- Since we assume unrestricted lending or borrowing at the risk-free rate (CAPM assumption 1), we obtain the following results.

Optimal Portfolio with A Risk-Free Asset

- If your optimal portfolio is at point T, then all your money is invested in the market portfolio of risky assets.
- If your optimal portfolio is to the left of T, you are putting some money in the risky portfolio and lending out the rest at the risk-free rate.
- If your optimal portfolio is to the right of T, you are borrowing at the risk-free rate and investing the money in the risky "tangency" portfolio.



Conclusion

- All investors will invest in the same risky portfolio T (the market portfolio).
- They will lend or borrow at the risk-free rate, depending on their risk preference, to attain their optimal.

Pricing Risky Assets

- Now we want to use the results we have obtained so far to show how to determine the risk premium and the price of risky assets.
- To determine the price of an asset, we need to know its required rate of return.
- But to find an asset's required rate of return, we need to know its risk premium.
- So we need to first find the "right" measure of risk for an individual asset: its systematic risk.

Measuring Systematic Risk

- The risk of an asset to an investor is the amount of risk that asset adds to the investor's portfolio risk.
- Since everyone invests in the market portfolio (which is completely diversified and therefore only has systematic risk), the only source of systematic risk of an asset is the systematic risk of the market portfolio.
- So how much risk an asset will add depends on how much [the return of] that asset "moves" with the market [return].

Measuring Systematic Risk

- Statistically, the added risk of an asset to an investor's portfolio risk is measured by its covariance with the market portfolio: $\sigma_{i,\mathrm{M}}$
- To measure the systematic risk of an asset **relative** to the market, we normalize (or standardize) the risk measure by dividing the covariance of an asset with the market portfolio by the variance of the market portfolio.

Measuring Systematic Risk

- We then have a risk measure called the **beta** of an asset:

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}$$

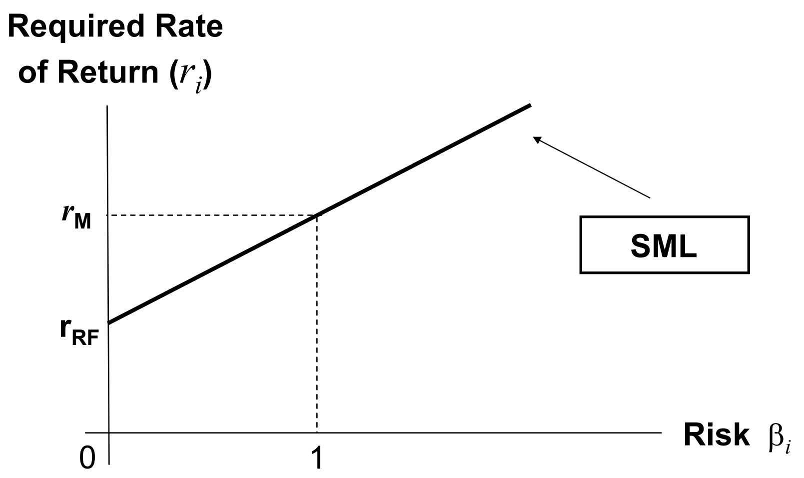
- That is, β_i measures the systematic risk in asset i relative to the risk of the market as a whole.
- Note: $\beta_M = \frac{\sigma_{m,m}}{\sigma_m^2} = \frac{\sigma_m^2}{\sigma_m^2} = 1$

The Security Market Line

- The required rate of return of an asset is the risk-free rate plus its risk premium.
- The risk premium of an asset is determined by its systematic risk (β_i) and the prevailing market risk premium ($r_M - r_{RF}$).
- The required rate of return of asset i is given by the Security Market Line

$$r_i = r_{RF} + \beta_i\,(r_M - r_{RF})$$

The Security Market Line (SML)



The meaning of β

- The SML $r_i = r_{RF} + \beta_i (r_M - r_{RF})$ can be re-written as

  $$\beta_i = (r_i - r_{RF}) / (r_M - r_{RF})$$

- If asset i is “riskier” than the market it will “commands” a higher risk premium than the market. Then $(r_i - r_{RF}) > (r_M - r_{RF}) \rightarrow \beta_i > 1$.
- If asset i is less risky than the market its risk premium will be lower than the market risk premium, i.e. $(r_i - r_{RF}) < (r_M - r_{RF}) \rightarrow \beta_i < 1$.

Estimating β

- To estimate β, we run a *linear regression* of the historical returns of the asset (r_i) on the historical returns of the market (r_M)

  $$r_{it} = \beta_i r_{Mt} + \alpha_i + \varepsilon_{it},$$

  where t is time period and ε_i is the error term.
- Using a calculator with a regression function or a spreadsheet program, we can estimate the coefficients $\alpha, \beta, \varepsilon$ of the regression line.
- Many analysts use the S&P500 as a proxy for the market return.

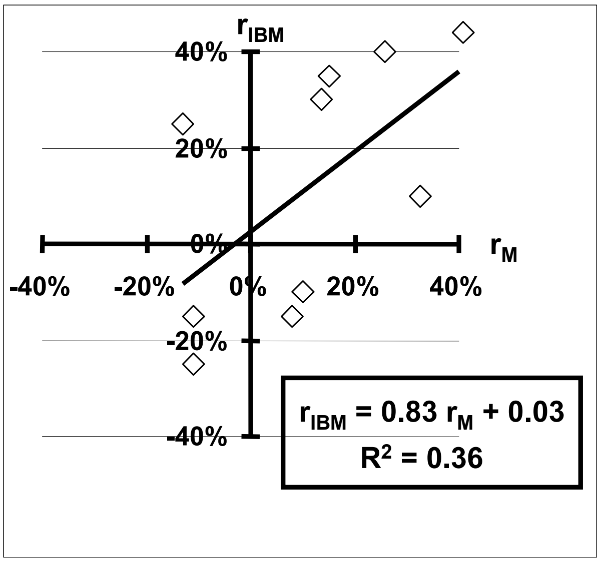
Estimating β

- Here $\beta_i r_M$ measures the systematic return while $\alpha_i + \varepsilon_i$ measures the non-systematic return of asset i.
- The β coefficient measures the sensitivity of the expected return of an asset to the variation of the market return.
- That is, β_i measures the systematic risk in asset i relative to the risk of the market as a whole.

Using historical stock returns to estimate the beta for IBM Stock.

Year	Market	IBM
1	25.7%	40.0%
2	8.0%	-15.0%
3	-11.0%	-15.0%
4	15.0%	35.0%
5	32.5%	10.0%
6	13.7%	30.0%
7	40.0%	42.0%
8	10.0%	-10.0%
9	-10.8%	-25.0%
10	-13.1%	25.0%

Example: Estimating Beta for IBM



Interpreting Regression Results

- The regression result of this example is:

  $r_{IBM} = 0.83\ r_M + 0.03$

- So the β coefficient for IBM is $\beta_{IBM} = 0.83$, which means if the market return increases by 1%, the required return on IBM stock will increase by 0.83%.

$$\beta_{IBM} = \frac{\Delta r_{IBM}}{\Delta r_M} = 0.83$$

Interpreting Regression Results

- The R^2 measures the "goodness of fit" or the percentage of a stock's variance that can be explained by the changes in market return.
- The typical R^2 is:
  - 0.3 for an individual stock
  - over 0.9 for a well-diversified portfolio

* The larger the value of R^2, the better the goodness of fit of the regression line.

Interpretation of β

From the regression line $r_i = \alpha_i + \beta_i r_M + \varepsilon_i$

$$\beta_i = \Delta r_i / \Delta r_M$$

- If $\beta_i > 1.0$, stock i is riskier than the market.
- If $\beta_i = 1.0$, stock i is of average risk.
- If $\beta_i < 1.0$, stock i is less risky than the market.

Most stocks have β's in the range of 0.5 to 1.5.

Using the SML to calculate the Required Return of an Asset

Suppose r_{RF} = 8%, r_M = 15%.

- The Market Risk premium is therefore

$$(r_M - r_{RF}) = 15\% - 8\% = 7\%.$$

- Using the SML:

$$r_i = r_{RF} + \beta_i (r_M - r_{RF})$$
$$= 8\% + \beta_i (7\%)$$

- To find the required return of an asset, all we need to do is to find its β!

The Larger the β the higher the required return

Notice that the larger the β of an asset, the higher its risk premium, and the higher its required rate of return. $r_i = r_{RF} + \beta_i (r_M - r_{RF})$

r_0 = 8.0% + (0.00)(15% - 8%) = 8.0%.

r_1 = 8.0% + (1.00)(15% - 8%) = 15.0%.

r_2 = 8.0% + (2.00)(15% - 8%) = 22.0%.

r_3 = 8.0% + (3.00)(15% - 8%) = 29.0%.

r_4 = 8.0% + (4.00)(15% - 8%) = 36.0%

Expected Return & Market Risk

We can estimate the β of each of the investment alternatives in our earlier example.

Security	Expected Return	Risk (β)
HT	17.4%	1.29
Market	15.0	1.00
USRE	13.8	0.68
T-bills	8.0	0.00
Repo	1.74	-0.86

Required Return of Alternatives

Using the SML, we can compute the required return of each alternative according to (predicted by) CAPM as follows:

r_{HT} = 8.0% + (1.29)(15% - 8%) = 17.0%
r_{M} = 8.0% + (1.00)(15% - 8%) = 15.0%.
r_{USRE} = 8.0% + (0.68)(15% - 8%) = 12.8%.
$r_{T\text{-}bill}$ = 8.0% + (0.00)(15% - 8%) = 8.0%.
r_{Repo} = 8.0% + (-0.86)(15% - 8%) = 2.0%.

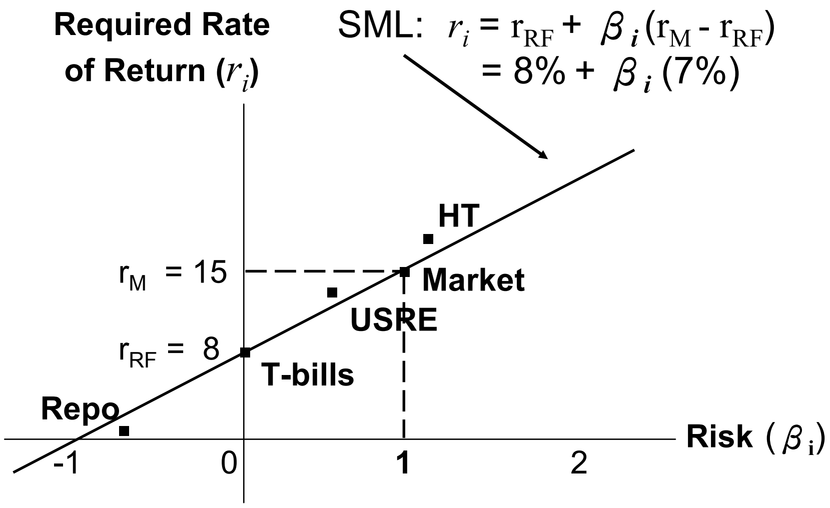
Required Return & Asset Pricing

- Once the required rate of return (r_i) of an asset is determined using the CAPM, we can use it to find the present value the asset's expected future cash flows to establish the "correct" price of the asset.
- In theory, an asset is correctly price if its observed price is the same as it value calculated using the required rate of return.

Expected vs. Required Returns

	Expected	Required	Asset is
H-T	17.4%	17.0%	Under-valued
Market	15.0	15.0	Fairly valued
USRE	13.8	12.8	Under-valued
T-bills	8.0	8.0	Fairly valued
Repo	1.74	2.0	Over-valued

SML: $r_i = r_{RF} + \beta_i (r_M - r_{RF})$
$= 8\% + \beta_i (7\%)$



SML and Investment Alternatives

Required Return & Asset Pricing

- If the expected return of an asset lies above the SML (greater than the required return of CAPM), it implies that the observed price of the asset is lower than its true value → under-valued (or under-priced) given its β.

The β and the Required Return of a Portfolio

Since the SML is a linear equation:

- The Required Return on a portfolio (r_p) is simply the required return of the assets in it multiplied by their portfolio weights!

  $$r_p = \Sigma\, w_i r_i$$

- The β of a portfolio (β_p) is simply the β's of the assets in the portfolio multiplied by their portfolio weights!

  $$\beta_p = \Sigma\, w_i \beta_i$$

Example 1: a portfolio with 60% HT and 40% Repo

- The Required Return of the portfolio:

  $$r_p = 0.6(r_{HT}) + 0.4(r_{Repo})$$
  $$= 0.6(17\%) + 0.4(2\%) = 11.0\%.$$

- The β of this portfolio:

  $$\beta_p = 0.6(\beta_{HT}) + 0.4(\beta_{Repo})$$
  $$= 0.6(1.29) + 0.4(-0.86)$$
  $$= 0.43$$

- Using the SML:

  $$r_p = r_{RF} + \beta_p(r_M - r_{RF})$$
  $$= 8.0\% + 0.43(7\%) = 11.0\%.$$

Example 2

- Suppose you are holding the following portfolio of stocks.

Stock	Investment	beta
A	$200,000	0.6
B	$200,000	1.0
C	$400,000	1.4
D	$400,000	1.8

- The risk-free rate is 6% and the required rate of return of the portfolio is 12.5%.

Example 2 (cont/...)

- What is the beta of this portfolio?

β_p = (2/12)0.6 + (2/12)1.0 + (4/12)1.4 + (4/12)1.8
= 16/12 = 4/3 (= 1.3333)

- What is the market risk premium?

$r_p = r_{RF} + \beta_p(r_M - r_{RF})$

0.125 = 0.06 + (4/3)MRP

MRP = 4.875%.

Example 2 (cont/...)

- If you sell all your holding of stock A and use the money to buy more stock D, what will be the beta of your new portfolio?

  $\beta_p = (2/12)1.0 + (4/12)1.4 + (6/12)1.8$
  $= 18.4/12 = 4.6/3\ (= 1.53333)$

- What will be the required return of the new portfolio?

  $r_p = r_{RF} + \beta_p(r_M - r_{RF})$
  $= 0.06 + (4.6/3)(4.875)$
  $= 13.535\%.$

Main Conclusions of The CAPM

- The relationship between risk and required return of an asset is linear and is given by the SML: $r_i = r_{RF} + \beta_i\,(r_M - r_{RF})$
  - The risk premium is determined by the systematic risk of the asset (β_i) and the prevailing market risk premium ($r_M - r_{RF}$).
- Portfolio risk (β_p) is simply the risk of the assets in the portfolio (β_i) multiplied by their portfolio weights (w_i).

Appealing Features of The CAPM

- The CAPM answers some of the fundamental questions in finance:
  - where to invest,
  - how to invest, and
  - what discount rate to use.

Empirical Test of The CAPM: The Stability of β Coefficients

- Betas of individual securities are unstable → past β not good estimators of future risk.
- Betas of portfolios with 10 or more randomly selected stocks are reasonably stable → past portfolio betas are good estimates of future portfolio volatility.
- Conclusion: CAPM is a good model for constructing investment portfolios but not for estimating the required returns of individual securities.

Empirical Test of The CAPM: The Slope of SML

Empirical tests show:

- A more-or-less linear relationship between realized returns and market risk.
- The slope is less than predicted by the SML: stocks with low β have higher return than the model predicts.
- Irrelevance of diversifiable risk specified in the CAPM model is questionable.
- Richard Roll questioned whether it was even conceptually possible to test the CAPM.

Conclusions regarding the CAPM

- It is impossible to verify or refute CAPM.
- Investors seem to be concerned with stand-alone (diversifiable) as well as market risk: the SML may not produce a correct estimate of r_i.
- The concepts of CML and SML are based on expectations, yet betas are calculated using historical data. Historical data may not reflect expectations about future riskiness.
- Other models may / will one day replace the CAPM, but it still provides a good framework for thinking about risk and return.

Arbitrage Pricing Theory (APT): A General Theory of Asset Pricing

- The CAPM is a single factor model because it assumes that the risk premium of an asset comes from only one source: market risk – the systematic risk of the market portfolio.
- The APT proposes that the relationship between risk and return is more complex and sensitive to multiple factors such as GDP growth, expected inflation, tax rate changes, and dividend yield.

Required Return for Stock *i* under the APT

- If there are k factors contributing to the risk premium of a stock i, the stock will be priced according to

$$r_i = r_{RF} + b_1(r_1 - r_{RF}) + b_2(r_2 - r_{RF}) + \dots + b_k(r_k - r_{RF}),$$

where

r_j = required rate of return on a portfolio sensitive only to economic Factor j.

b_j = factor-specific *beta*: the sensitivity of Stock i to economic Factor j,

for all $j = 1, \dots, k$

Required Rate of Return under the APT

- If asset prices diverge from the required rate of return implied by the model, arbitrage will restore them back to the equilibrium prices.
- Under APT, different systematic risks may require different risk premium.
- For example, if long-term interest rates have been stable for many years, there should be no risk premium for holding interest-sensitive assets or securities.

What is the status of the APT?

- The APT is being used for some real world applications.
- Its acceptance has been slow because the model does not specify what factors influence stock returns.
- More research on risk and return models is needed to find a model that is theoretically sound, empirically verified, and easy to use.

Fama-French 3-Factor Model

- In additional to the "excess market return" ($r_M - r_{RF}$) used in CAPM as the determinant of the required return of an asset, Eugene Fama and Kenneth French propose a three-factor model by adding two other factors.
- Fama and French start with the observation that two classes of stocks earn higher return relative to the market than predicted by their beta:
  1. Small-cap Stocks
  2. Value Stocks (stocks with high Book to Market Value ratio).

Fama-French 3-Factor Model (Continued)

1. r_{SMB}, the "S minus B" (excess) return:
   where S is the return on a portfolio of small firms (where size is based on the market value of equity) and B is the return on a portfolio of big firms.
2. r_{HML}, the "H minus L" (excess) return:
   where H is the return on a portfolio of firms with high book-to-market value ratios (using book equity and market equity values) and L is the return on a portfolio of firms with low book-to-market value ratios.

Required Return for Stock *i* under the Fama-French 3-Factor Model

$r_i = r_{RF} + b_i(r_M - r_{RF}) + c_i(r_{SMB}) + d_i(r_{HML})$

where
b_i = sensitivity of Stock *i* to the market return.
c_i = sensitivity of Stock *i* to the size factor.
d_i = sensitivity of Stock *i* to the book-to-market factor.

Required Return for Stock *i* Example

- Suppose we have the following estimates:
  - $b_i = 0.9$, $c_i = -0.5$, $d_i = -0.3$,
  - $r_{RF} = 6.8\%$
  - the market risk premium $r_M - r_{RF} = 6.3\%$,
  - the expected value for the size factor $r_{SMB} = 4\%$,
  - the expected value for the book-to-market factor $r_{HML} = 5\%$.

Required Return for Stock *i* CAPM vs. Fame-French 3-Factor Model

CAPM (single factor):

$r_i = r_{RF} + b_i(r_M - r_{RF})$
$= 6.8\% + (0.9)(6.3\%)$
$= 12.47\%$

Fama-French (3 factors):

$r_i = r_{RF} + b_i(r_M - r_{RF}) + c_i(r_{SMB}) + d_i(r_{HML})$
$= 6.8\% + (0.9)(6.3\%) + (-0.5)(4\%) + (-0.3)(5\%)$
$= 8.97\%$

- In this example, the required return estimated by the 3-factor model is lower than that of the CAPM.