Lecture 4:
Risk and Return

- Basics of Investment Return
- Basics of Investment Risk
  - Variance / standard deviation of return
  - Coefficient of Variation

Introduction

- All investments can be characterized by two basic features:
  - Risk
  - Return
- We will devote this lecture is to these two concepts.
- It is important to understand how risk is defined and measured because the appropriate return of an investment depends on how "risky" it is.

Introduction

- We will proceed in two steps:
  - first we will introduce the concepts of risk and return using **historical** data,
  - then we will discuss how to define and measure **future** (anticipated) return and risk.

What are investment returns?

- Investment returns
  - measure the financial results of an investment.
  - are typically expressed in percentage terms – rates of return.
- There are
  - historical (realized) returns
  - expected (future or anticipated) returns

Defining Rate of Return

- Let P_t be the purchase price of a share of stock (or a coupon bond) at time t and P_{t+1} be its price in one year.
- The rate of return is:
  - If no dividend is paid:

    $R_t = (P_{t+1} – P_t) / P_t$
  - If dividend D_t (or coupon C_t) is paid:

    $R_t = (P_{t+1} – P_t + D_t) / P_t$

A Simple Example

Suppose you purchased a 6% coupon bond for $980 and sold it for $1,015 after 1 year.

What is your return on this investment?

- In dollar terms:

  $ Received + Coupon – $ Invested

  $1,015 + $60 – $980 = $95.
- In percentage terms (rate of return):

  $ Return / $ Invested

  $95 / $980 = 0.0969 or 9.69%.

Defining and Measuring Risk

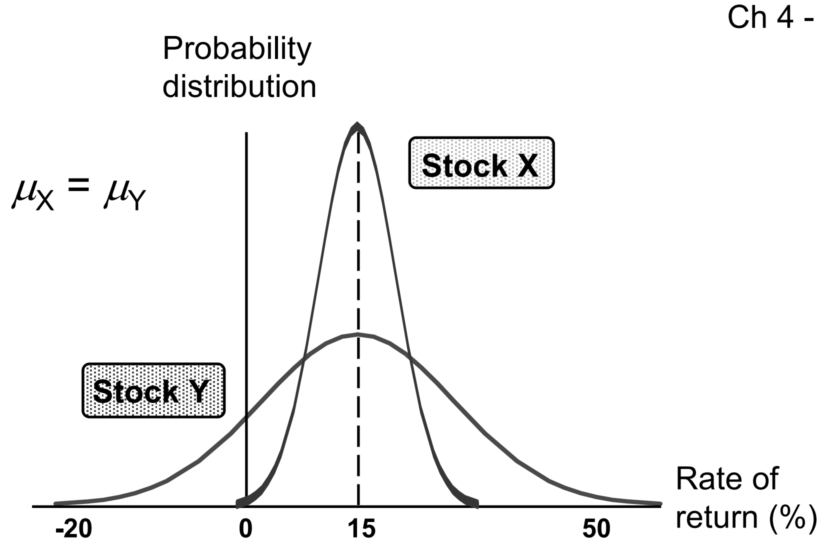
- We will analyze risk in two steps:
  1. Define risk in terms of
     - variability of realized returns
     - uncertainty of future returns.
  2. Decompose risk into *diversifiable risk* and *non-diversifiable risk*. (next lecture)

What is Investment Risk?

- Risk, for most of us, means something negative.
- In finance, investment risk refers to *the probability that the return on an investment is different from that of expected.*
- Thus, investment risk includes "good" (higher than expected) as well as "bad" (lower than expected) outcomes.
- The greater the chance that the actual return is different from the expected return, the greater the risk.

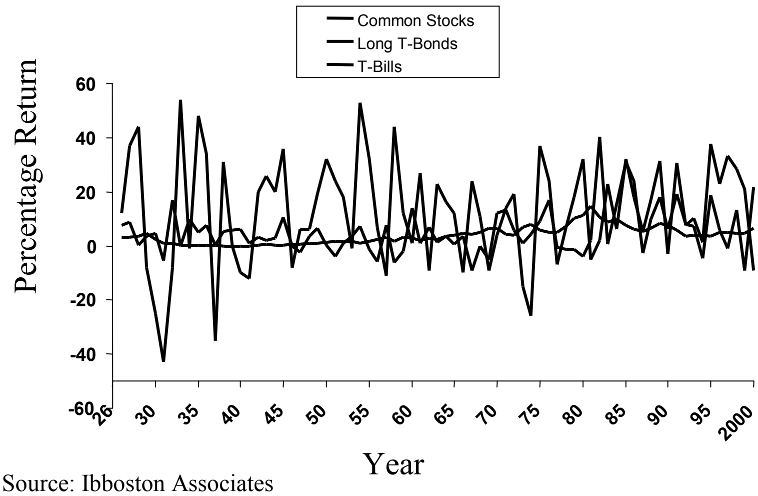
How to measure risk?

- One common measure of investment risk is the standard deviation (s.d.) of returns, which measures the "spread" of returns around the mean (or average) return.
- The larger the variability of returns – "spread" around the average – the higher the risk.
- Consider the probability distributions of returns on two stocks, X and Y.
  - Both have an average return of 15%
  - Which one has more risk?



- Which stock is riskier? Why?

Rates of Return 1926-2000



Source: Ibboston Associates

Computing Historic Risk & Return

- The average return over *T* periods is simply the arithmetic <u>mean</u> of returns:

$$\bar{r} = \frac{1}{T}(r_1 + r_2 + \cdots + r_T)$$

where r_i is the rate of return in period i.

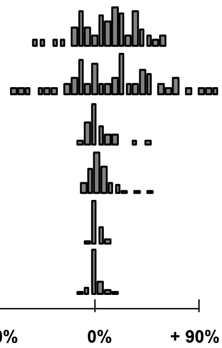
- The Standard Deviation (s.d.) of returns is the "spread" of returns around the mean:

$$SD(r) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - \bar{r})^2}$$

$$= \sqrt{\text{var}(r)}$$

Historical Returns, 1926-2002

Series	Average Annual Return	Standard Deviation	Distribution
Large Company Stocks	12.2%	20.5%	
Small Company Stocks	16.9	33.2	
Long-Term Corporate Bonds	6.2	8.7	
Long-Term Government Bonds	5.8	9.4	
U.S. Treasury Bills	3.8	3.2	
Inflation	3.1	4.4	



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Risk Premium

- Notice the risk-return tradeoff in different kinds of financial assets: those with higher average returns also carry higher risk than those with lower average returns.
- The average **excess** return on a risky asset over the return on a safe investment is called a risk premium

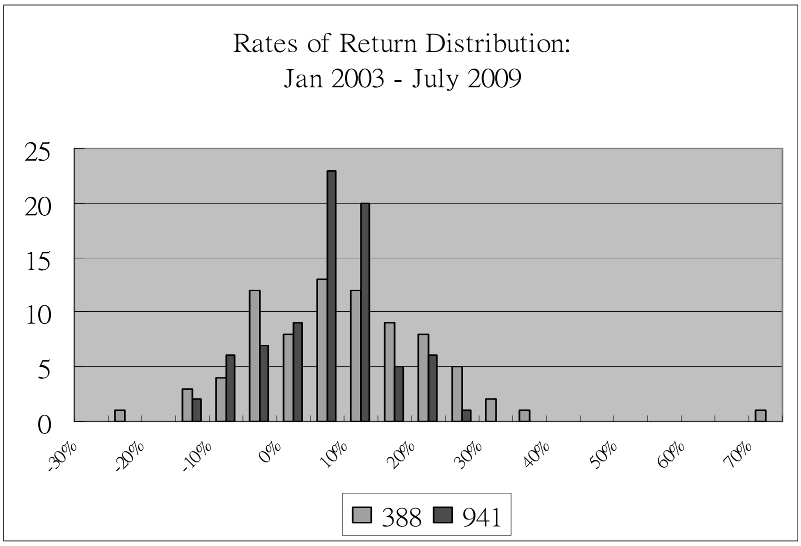
  Risk Premium = Average Return on Risky Asset – Return on Safe Asset

- T-Bills are generally considered a risk-free asset.

Monthly Rate of Return: January 2003 – July 2009 (in %)

Stock	Avg.	s.d.	High Return	Low Return
HKEX (0388)	4.81	13.95	68.03	-27.08
China Mobile (0941)	2.58	8.79	24.37	-16.90

The graph in the next slide shows the frequency distributions of the monthly rates of return of the two stocks during that period.



Expected Future Returns

- What we have looked at are historical returns and their variability (or risk).
- When evaluating investment choices, we care about their expected future returns.
- Typically, investment returns are not known with certainty in advance because future cash flow streams are not guaranteed.
- To capture the uncertainty of future events, we use concepts from probability and statistics.

Expected Value & Standard Deviation of Return

- The future (expected) return of an asset is a random variable, the value of which depends on the possible "states of the world".
- The Expected Return of an asset is the mean (or average value of) returns weighted by the probability of all possible states of the world.
- The Standard Deviation of return – the variability of the actual return from the expected return – is a measure of the riskiness (or volatility) of an asset.

Review of Statistical Concepts

- Random Variable
- States of the world
- Expected Value
- Variance and Standard Deviation
- Covariance
- Correlation

Review: Random Variables

- A **random variable** is a variable whose possible values are *random* (i.e. not deterministic) but its statistical distribution is known.
- Example:
  - The outcome of rolling a dice is a random variable because the outcome of each roll belongs to the set of possible values {1, 2, 3, 4, 5, 6}.
  - If it is a "balanced" dice, all possible outcomes are equally likely to occur with probably 1/6.

Review: States of the World

Example:

- Let X be a random variable whose value depends on three possible States of the World (for example, "Sunny", "Cloudy", and "Rain")

  s = {s_1, s_2, s_3}

  with probabilities

  $p = \{p_1, p_2, p_3\}$,

  where $\Sigma_i\, p_i = p_1 + p_2 + p_3 = 1$.
- That is, $X = x_i$ with probability p_i, for $i = 1, 2, 3$.

Review: Interpretation

X can take on three possible values $\{x_1, x_2, x_3\}$, depending on the states of the world:

- The value of X will be x_1 with probability p_1 if state s_1 occurs.
- The value of X will be x_2 with probability p_2 if state s_2 occurs.
- The value of X will be x_3 with probability p_3 if state s_3 occurs.

Review: Expected Value

- The **Expected Value** (or mean) of *X* gives a measure of the center of the distribution of *X* and is defined as

$$\mu_x = E(X) = \sum_i p_i x_i$$

- So *E(X)* is simply the weighted average of *X* – the average of the possible values of *X* weighted by their probabilities.
- In the example above, the expected value of *X* is

$$\mu_x = p_1 x_1 + p_2 x_2 + p_3 x_3$$

Review: Variance & Standard Deviation

- The **Variance** of *X* is a measure of the spread of the distribution about the mean and is defined by

$$\sigma_x^2 = E(X - E(X))^2$$
$$= E(X - \mu_x)^2$$

- The **Standard Deviation** of *X* is simply the square root of its variance

$$\sigma_x = \sqrt{\sigma_x^2}$$
$$= \sqrt{E(X - \mu_x)^2}$$

Review: Variance & Standard Deviation

In the example above:

- The **Variance** of X is

$$\sigma_x^2 = E(X-\mu_x)^2 = \sum_i p_i(x_i-\mu_x)^2$$

$$= p_1(x_1-\mu_x)^2 + p_2(x_2-\mu_x)^2 + p_3(x_3-\mu_x)^2$$

- The **Standard Deviation** of X is simply

$$\sigma_x = \sqrt{\sigma_x^2}$$

$$= \sqrt{p_1(x_1-\mu_x)^2 + p_2(x_2-\mu_x)^2 + p_3(x_3-\mu_x)^2}$$

Review: Covariance and Correlation

- The Covariance of two random variables X and Y measures how much two variables vary together.

$$\sigma_{xy} = E(X-E(X))(Y-E(Y))$$

$$= \sum_i p_i(x_i-\mu_x)(y_i-\mu_y)$$

- The correlation coefficient, ρ, measures the degree to which the values of X and Y vary linearly

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

ρ is pronounced “rho”.

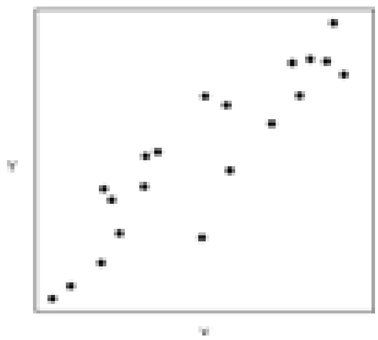
Review: The Correlation Coefficient

The value of ρ must lie between 1 and -1. That is

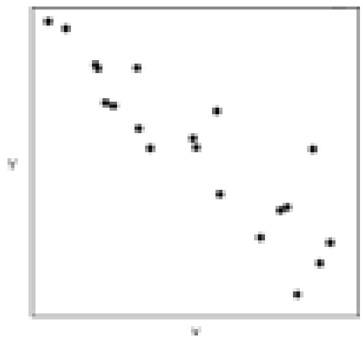
$$-1 \leq \rho \leq 1$$

- If $\rho_{xy} > 0$, X and Y are positively correlated.
  If $\rho_{xy} = 1$, X and Y are perfectly positively correlated.
- If $\rho_{xy} < 0$, X and Y are negatively correlated.
  If $\rho_{xy} = -1$, X and Y are perfectly negatively correlated.
- If $\rho_{xy} = 0$, X and Y are uncorrelated.

$\rho > 0$: X and Y are positively correlated



$\rho < 0$: X and Y are negatively correlated



If $\rho = 1$ (= -1) the dots will lie on an upward (downward) sloping straight line

A More Concrete Example

Computing mean and s.d. under uncertainty.

- Suppose the economy has 3 possible “states" -- *recession*, *average*, and *boom*.
- The probability of each state and rate of return on Stocks *A* and B are estimated as follows.

State	Prob.	r_A	r_B
Recession	0.2	4%	18%
Average	0.6	12%	8%
Boom	0.2	20%	2%

The μ and σ of Stock A

- Expected Return of Stock *A*

$$\mu_A = (0.2)(0.04) + (0.6)(0.12) + (0.2)(0.2)$$
$$= 12\%$$

- Standard Deviation of Return of Stock *A*

$$\sigma_A = [\ (0.2)(0.04-0.12)^2 + (0.6)(0.12-0.12)^2 + (0.2)(0.2-0.12)^2\ ]^{1/2}$$
$$= (0.00256)^{1/2}$$
$$= 0.0506 = 5.06\%$$

The μ and σ of Stock B

- Expected Return of Stock *B*

$\mu_B = (0.2)(0.18) + (0.6)(0.08) + (0.2)(0.02)$

$= 8.8\%$

- Standard Deviation of Return of Stock *B*

$\sigma_B = [\ (0.2)(0.18-0.088)^2 + (0.6)(0.08-0.088)^2 + (0.2)(0.02-0.088)^2\ ]^{1/2}$

$= (0.00266)^{1/2}$

$= 0.0515 = 5.15\%$

Interpretation

- The expected (or average) return on Stock *A* is 12%, with a variability of 5.06%.
- The actual return (within 1 s.d.) can be
  - as high as 17.06% (= 12% + 5.06%)
  - as low as 6.94% (= 12% - 5.06%)

  with positive probabilities.
- (same applies to Stock B)

Covariance and Correlation

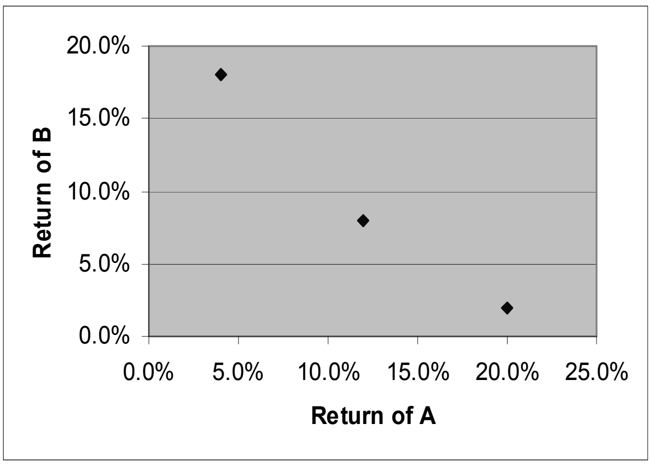
- Covariance of *A* and *B*

$$\sigma_{AB} = (0.2)(0.04-0.12)(0.18-0.088) + (0.6)(0.12-0.12)(0.08-0.088) + (0.2)(0.20-0.12)(0.02-0.088) = -0.00256$$

- Correlation Coefficient

$$\rho_{AB} = \sigma_{AB} / \sigma_A\sigma_B = -0.00256 / (0.0506)(0.0515) = -0.9818$$

Correlation of r_A and r_B



Interpretation

- The **covariance** of *A* and *B* is negative → their rates of return vary in opposite direction.
- Note that the plots of r_A and r_B lie roughly on a *downward sloping* straight line →
  - The rates of return of *A* and *B* are almost *perfectly negatively correlated.*
  - i.e. The **correlation coefficient** $\rho \approx -1$
- We will look at an elaborate example below.

Consider the Hypothetical Returns of the Following Investment Alternatives

Economy	Prob.	T-Bill	Hi-Tech	Repo	US Real Estate	Mkt Portfolio
Recession	0.10	8.0%	-22.0%	28.0%	10.0%	-13.0%
Below avg.	0.20	8.0	-2.0	14.7	-10.0	1.0
Average	0.40	8.0	20.0	0.0	7.0	15.0
Above avg.	0.20	8.0	35.0	-10.0	45.0	29.0
Boom	0.10	8.0	50.0	-20.0	30.0	43.0
	1.00					

Overview of the data

- The return on the market portfolio (S&P500 is usually used as a proxy) is our benchmark.
- The return on T-bills is 8% regardless of the state of the economy ("risk-free" rate of return).
- Hi-Tech moves **with** the economy, so its return is positively correlated with that of the economy. This is a typical situation.
- Repo moves **counter to** the economy. Such negative correlation is atypical.
- We want to find the μ and σ of each alternative.

The Expected Rate of Return (μ) on each alternative

Prob$_i$ = probability of state i occurring

r_i = rate of return if state i occurs

$$\mu = \sum_{i=1}^{n} Prob_i \cdot r_i .$$

The expected return of Hi-Tech:

$$\mu_{HT} = 0.10\,(-0.22) + 0.20\,(-0.02) + 0.40\,(0.20) + 0.20\,(0.35) + 0.10\,(0.50) = 0.174 = 17.4\%.$$

Do the same for all the alternatives to obtain:

	μ
Hi-Tech	17.4%
Market	15.0%
USRE	13.8%
T-bill	8.0%
Repo	1.74%

- Hi-Tech has the highest expected rate of return.
- Does that make it the best choice?

The Standard Deviation of Return (σ) for each alternative

Formula for computing Standard Deviation

$$\sigma = \text{Standard deviation}$$

$$\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$$

$$= \sqrt{\sum_{i=1}^{n} Prob_i \left(r_i - \mu_i \right)^2}.$$

Hi-Tech $\sigma_{HT} = (0.10(-0.22 - 0.174)^2 + 0.20(-0.02 - 0.174)^2 + 0.40(0.20 - 0.174)^2 + 0.20(0.35 - 0.174)^2 + 0.10(0.50 - 0.174)^2)^{1/2}$

$= 0.04^{1/2}$

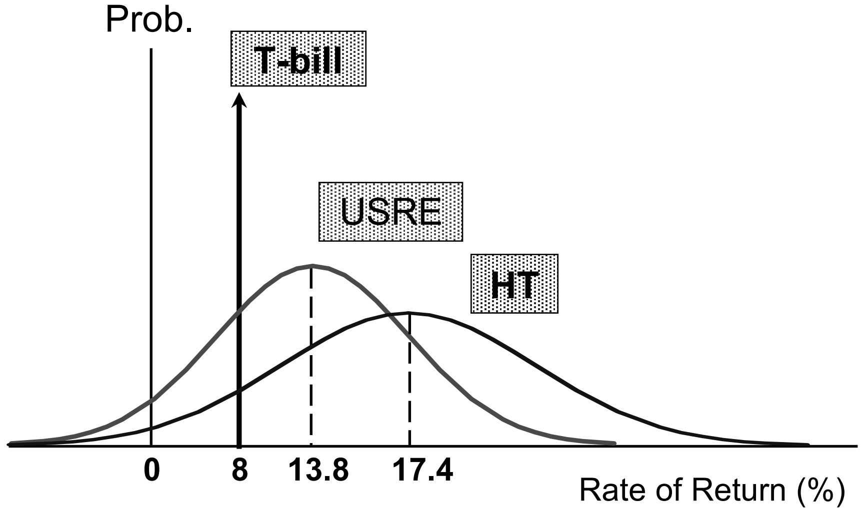
$= 0.20$ or 20.0%.

$\sigma_{T\text{-bills}} = 0.0\%$.
$\sigma_{HT} = 20.0\%$.

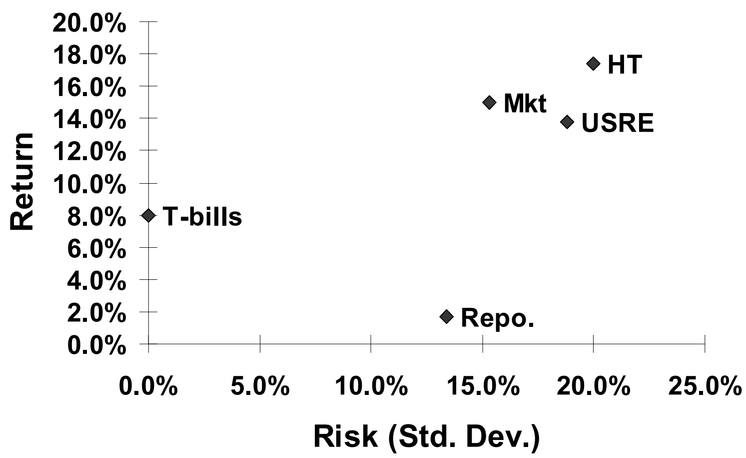
$\sigma_{Repo} = 13.4\%$.
$\sigma_{USRE} = 18.8\%$.
$\sigma_{M} = 15.3\%$.

Summary of Expected Return & Risk

Security	Expected Return (μ)	Risk (σ)
Hi-Tech	17.40%	20.0%
Market	15.00%	15.3%
USRE	13.80%	18.8%
T-bills	8.00%	0.0%
Repo	1.74%	13.4%



Return and Risk of each alternative



Which alternative is the "best" choice?

- If you could invest in only one of these 5 alternatives, which one is "the best"?
- To answer this question:
  - We need to know how to compare the risk of different alternatives which have different expected returns?
  - We need to know you risk preference – How much risk are you willing to take?

Stand-alone Risk

- The Standard Deviation of return (σ) measures the risk of an investment.
- How do we compare the risk of investment alternatives with different expected returns?
- To compare the risk (variation of returns) of alternatives with different expected (mean) returns, we use the Coefficient of Variation

$$CV = \sigma/\mu.$$

i.e. its "risk per unit of expected return".

Coefficient of Variation (CV): = Standard deviation / expected return

- CV measures the risk (variation of return) per unit of return of investment alternatives.

$CV_{T\text{-}Bills}$ = 0.0% / 8.0% = 0.0

CV_{HT} = 20.0% / 17.4% = 1.1

CV_{Repo} = 13.4% / 1.74% = 7.9

CV_{USRE} = 18.8% / 13.8% = 1.4

$CV_{Market\ Port.}$ = 15.3% / 15.0% = 1.0

Summary: μ, σ and CV of Different Alternatives

Security	Expected Return (μ)	Risk (σ)	CV
Hi-Tech	17.4%	20.0%	1.1
Market	15.0%	15.3%	1.0
USRE	13.8%	18.8%	1.4
T-bills	8.0%	0.0%	0.0
Repo	1.74%	13.4%	7.9

Back to our earlier question

- Questions:
  - T-Bills, which is essentially risk-free, has the lowest CV. Does it mean it is the "best" investment alternative?
  - By the same token, Repo has the highest CV, but does it mean it is the "worst" investment alternative to take?
- The answer clearly depends in part on the risk preferences of the investor.
- But the questions assume you can only choose one investment alternative.
- A more meaningful question is: What combination of investments is the best?
- To answer this question, we need to take a "deeper" look at the notion of risk (in the next lecture).

Risk & Diversification

- The two measures of risk we have introduced so far – σ and CV – are both stand-alone risk: the risk of a security if held in isolation.
- The risk of a security can actually be decomposed into:
  - Diversifiable Risk: Specific (or Unique) Risk which can be diversified away.
  - Non-diversifiable Risk: Market Risk which is not diversifiable.

Diversification

- When economic conditions change, some assets will rise in value while others will decline in value.
- By forming a portfolio and spreading their investment over a variety of assets, investors can lower the overall level of risk
- Old Wisdom: “Don’t put all your eggs in the same basket”.

* A portfolio = a collection of assets or securities.

Summary

Defining and measuring risk

- Risk is defined in terms of uncertainty about future returns.
- It is commonly measured by σ, the standard deviation or volatility of returns.
- To compare the “risk” of investments with different expected returns, we use the coefficient of variation: CV = σ / μ
- σ and CV both measure the stand-alone risk of an asset.

Next Lecture

- Decompose risk into *diversifiable* risk and *non-diversifiable* risk.
- Theory of Portfolio Choice: given a set of assets, how do we find the optimal portfolio given the investor's risk preference.