International Business Economics

Lecture Notes
Set #4

Game Theory & Strategic Thinking

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Chapter 1

Game Theory

1.1 Introduction

- Game theory is a branch of mathematics invented in the 40's by John von Neumann. *Theory of Games and Economic Behavior* (with Oskar Morgenstern), 1944.
- A game or rivalry situation arises whenever the actions of one party may affect the well-being of another. So each party must take into account the action of another party when making its decision.
- Game theory is a systematic way to analyze strategic behavior in rivalry situations and try to predict the outcome.
- Fun Games: rock-scissors-paper, matching pennies, poker, blackjack, chess.

  Business / Economic Games: product choice, pricing, advertising campaigns, entry deterrence, trade war

  Political Games: arms race, lobbying, negotiations, voting, trade war

1.2 Cooperative Vs. Non-cooperative Games

- We will be concerned with non-cooperative games only.
- "Non-cooperative" does not mean that the players don't get along or that they refuse to cooperate.
- Non-cooperative games differ from cooperative games in that players are either not allowed or not able to make binding agreements before they play the game.
- There are many business / economic situations in which the parties involved either cannot enter any legal agreement or that the agreements are not binding because they are impossible or too costly to enforce.
- One of the objectives of non-cooperative game theory is to explain HOW, and under WHAT CONDITIONS, can cooperation emerge from self-interest individual behavior within a given set of rules.

1.3 The Prisoners' Dilemma

The best way to learn game theory is to work through some examples. So before we introduce some of the important concepts, let's start with an example of a class of game that has many business applications – The Prisoners' Dilemma.

1.3.1 A Classic Version

- Two criminals are arrested but the police don't have enough evidence to convict them for the crime they jointly committed.
- If they both keep silent and not confess, the police can only charge them with some minor infraction that carries a 6-month sentence.
- If they both confess, then the police can charge them with the crime and they can be sentenced to 3 years in jail.
- The police keep the two in separate cells and offer each one the following choices:
  - If one of them is willing to confess and implicate the other, then he gets to go free but his accomplice will get a 10-year sentence.
  - If he does not confess but his accomplice does, then he will get 10 years while his partner will go free.
- So each player has two strategies to choose from: *Confess* or *Not Confess*, and there will be four (4) strategy combinations between the two players.
- The following payoff matrix summarizes all the possible outcomes of this game, where in each cell the first number is the payoff to Player A and the second number is the payoff to Player B.

		Player B	
		Confess	Not Confess
Player A	Confess	-3 yrs , -3 yrs	go free , -10 yrs
	Not Confess	-10 yrs , go free	-6 mths , -6 mths

- How would the players choose? and what will be the likely outcome?

Player A: "Suppose B chooses *Confess*. If I confess I will get 3 years, but if I don't I will get 10 years. So I should choose *Confess*."

"Suppose B chooses *Not Confess*. If I confess I get to go free, but if I don't I will get 6 months. So I should choose *Confess*."

Conclusion: A should choose *Confess* no matter what B chooses.

Player B: Since the payoffs are symmetric, the same logic applies to Player B.

Conclusion: B should choose *Confess* no matter what A chooses.

- Equilibrium outcome: $\{$*Confess, Confess*$\}$, and they both get 3 years in the slammer.
- What is so interesting about this game is that

1. It is quite apparent how any intellectual individual will play this game: no matter what your opponent will do, it is in your best interest to *Confess*

   The "dilemma" is that even both players know that they can get a "better" payoff by cooperating, there is a strong incentive to act selfishly and end up with a sub-optimal outcome.

2. We can replace *Not Confess* with *Cooperate* (contribute to the common good) and *Confess* with *Defect* (act selfishly), and apply the game to many business situations.

3. The outcome of the game can change significantly if it is repeated. Specifically, repetition opens up the possibly for punishment and retaliation that can help enforce cooperation.

1.3.2 A Stylized Version

		Firm B	
		Cooperate	Defect
Firm A	Cooperate	$3, 3$	$0, 5$
	Defect	$5, 0$	$1, 1$

The numbers in each cell are the payoffs (profits) to Firm A and Firm B respectively.

1.4 The Basics

1.4.1 Basic Elements of A Game

The basic elements of a game are:

- A set of players

- A set of actions or strategies for each player to choose from

- Protocol ("rules of the game"): what kind of move is allowed, order of play (simultaneous vs. sequential move), how the game ends, and the information structure (who knows what and when)

- A set of payoffs associated with each possible outcome

  An outcome of a game is a strategy profile: a set of strategies, one from each player.

1.4.2 Equilibrium Concepts

- The equilibrium of a game is defined in terms of a strategy profile – a combination of strategies, one from each player – and not its associated set of payoffs.

  The equilibrium of the Prisoners' Dilemma game above is {*Confess, Confess*} and <u>not</u> (-3 yrs, -3 yrs).

- The Nash criterion (named after John Nash of "A Beautiful Mind") for an equilibrium is that each player chooses the best response – strategy that maximizes her own payoff – given the supposed strategies of the other players.

- A **Nash Equilibrium (NE)** is a strategy profile or combination in which the strategy of every player is a best response to all the other players' strategies in the profile.
- A NE is **self-enforcing** because there is no incentive for any player to deviate from his/her strategy unilaterally, and therefore is optimal for all the players to stick with their own strategy in the profile.
- It is easy to check that {*Confess, Confess*} is self-enforcing. Given Player B's strategy in the profile (*Confess*), Player A has no incentive to deviate and switch to *Not Confess*. Same for Player B.

** If a game has an "obvious solution," then that solution must be Nash. But a game have may have no obvious solution or it may have more than one Nash equilibrium.

1.5 Dominance

- In simple games, sometimes we can spot the NE simply by inspection.

		B	
		t_1	t_2
A	s_1	$6, 6$	$2, 3$
	s_2	$3, 2$	$1, 1$

This game has a unique Nash Equilibrium: $\{s_1, t_1\}$

- But this one is not so obvious.

		B	
		t_1	t_2
A	s_1	$2, 7$	$0, 4$
	s_2	$5, 1$	$-4, -2$

We can check every single strategy combination to see which one(s) satisfies the Nash criterion for an equilibrium. But this is not always feasible – if each player has 4 strategies to choose from, we will have to check 16 strategy profiles!

- So we need some systematic ways to solve a game (i.e. find the NE, or try to narrow down the number of possible outcomes if there is more than one equilibrium).
- We say a player has a **dominant strategy** if there is one strategy that gives uniformly higher payoffs than all other strategies no matter what the other players do.
- Consider the stylized version of the Prisoners' dilemma above, looking from Player A's perspective.

		Player B	
		β_1	β_2
Player A	α_1	$3, -$	$0, -$
	α_2	$5, -$	$1, -$

If Player B chooses β_1, Player A is better off choosing α_2 (because $5 > 3$). And if B chooses β_2, A is better off choosing α_2 also (because $1 > 0$). So α_2 is Player A's dominant strategy.

- Now let's look at the game from Player B's perspective.

		Player B	
		β_1	β_2
Player A	α_1	$-, 3$	$-, 5$
	α_2	$-, 0$	$-, 1$

  Player B is better off playing β_2 regardless of what Player A does because $5 > 3$ and $1 > 0$. So β_2 is Player B's dominant strategy.

- A **Dominant Strategy Equilibrium (DSE)** is a strategy profile in which all the players play their dominant strategy (provided that they <u>all</u> have one, of course).

- The Prisoner's Dilemma is a type of game in which every player has a dominant strategy. Each player acting on their own interest will play their dominant strategy and result in a less desirable outcome. **The Dilemma** is that they are all better off (get higher payoffs) if they can agree to play cooperatively <u>and</u> abide by the agreement.

- In a simple game where each player only has two strategies to choose from, if a **dominant** strategy exists the other one must be a **dominated** strategy.

- We say "strategy α_1 is strictly dominat**ed** by strategy α_2" for a player if α_2 gives the player strictly higher payoffs than α_1 does, **regardless** of the strategy choices of his/her opponents.

- **Dominance tells us what will NOT happen: strategies that are <u>dominated</u> will not be played by a rational player**.

- A game may <u>not</u> have a dominant strategy (nice if it does, but too much to ask), but it may have strategies that are dominated by another. To solve a game, we can start by eliminating any dominated strategies – strategies that a player should not play.

- Consider the following game. Player 2 does not have a dominant strategy, but L is strictly dominat<u>ed</u> by R. No matter what Player 1 does, he should NOT play L.

		Player 2		
		L	M	R
Player 1	T	$3, 2$	$2, 1$	$1, 3$
	B	$1, 0$	$3, 6$	$2, 1$

  Does Player 1 have a dominant / dominated strategy? Ans: No.

- So we can eliminate strategy L of Player 2 and the game can be reduced to

		Player 2		
			M	R
Player 1	T		$2, 1$	$1, 3$
	B		$3, 6$	$2, 1$

  because Player 1 knows that 2 will not play L is he is rational.

- In this "reduced" game, B is Player 1's dominant strategy (which makes T a dominated strategy).

- This game can be further reduced and leaves $\{B, M\}$ as the NE.

		Player 2	
		M	R
Player 1			
B		$3,6$	$2,1$

1.6 2x2 Toy Games

Consider the following 2 by 2 games: games played between 2 players, each has 2 strategies. They are all simultaneous move games in which the players have to make their choice of play without knowing what their opponents will choose.

Use these "toy games" to practice finding Nash equilibria and solving a game. The answers and their explanations are provided in the next section

1.

A \ B	t_1	t_2
s_1	$6,6$	$2,3$
s_2	$3,2$	$1,1$

2.

A \ B	t_1	t_2
s_1	$2,7$	$0,4$
s_2	$5,1$	$-4,-2$

3.

A \ B	t_1	t_2
s_1	$2,7$	$0,4$
s_2	$5,1$	$-4,1$

4.

A \ B	t_1	t_2
s_1	$5,5$	$-3,8$
s_2	$8,-3$	$0,0$

5.

A \ B	t_1	t_2
s_1	$10,10$	$0,0$
s_2	$0,0$	$1,1$

6.

A \ B	t_1	t_2
s_1	$10,0$	$5,2$
s_2	$10,1$	$2,0$

7.

		B t_1	B t_2
A	s_1	$0, 0$	$81, 80$
A	s_2	$80, 81$	$0, 0$

8.

		B t_1	B t_2
A	s_1	$1, -1$	$-1, 1$
A	s_2	$-1, 1$	$1, -1$

9.

		B t_1	B t_2
A	s_1	$3, 1$	$1, 3$
A	s_2	$0, 5$	$4, 2$

10.

		B t_1	B t_2
A	s_1	$-3, -3$	$2, 0$
A	s_2	$0, 2$	$1, 1$

1.6.1 Solutions to Toy Games

1. This game has a unique Nash Equilibrium $\{s_1, t_1\}$. It is also a DSE.

2. This game has a unique Nash Equilibrium $\{s_2, t_1\}$. Player B has a dominant strategy t_1 but Player A does not have one. Apply strict dominance and eliminate t_2 because it will not be played by player B. Given that it is rational for player B to play t_1, player A's best response (to t_1) is s_2. Here we apply strict dominance iteratively, first to player B then to player A, to solve this game.

3. This is similar to the game above, except that we allow for "weak dominance".

   Strategy t_1 weakly dominates t_2 for player B because t_1 is "at least as good as" t_2 in terms of payoffs. If A plays s_1, Player B is strictly better off playing t_1 (because $7 > 4$). If A plays s_2, Player B is indifferent between playing t_1 and t_2 because he will get the same payoff either way. If we apply weak dominance to this game, the unique Nash equilibrium of this game is $\{s_2, t_1\}$.

4. This is a Prisoner's Dilemma with a unique Nash equilibrium $\{s_2, t_2\}$ and a payoff of (0,0). The NE is stable but sub-optimal because both players are better off with a higher payoff of (5,5) if they act cooperatively and play $\{s_1, t_1\}$ instead.

   The cooperative outcome is "unstable" because both players have an incentive to defect and play their dominant strategy (cooperating is strictly dominated by defecting). So the NE is also a DSE.

5. This game has two Nash Equilibria: $\{s_1, t_1\}$ and $\{s_2, t_2\}$. The "obvious" solution is $\{s_1, t_1\}$.

6. There are two Nash equilibria in this game: $\{s_1, t_2\}$ and $\{s_2, t_1\}$. But note that s_2 is weakly dominated by s_1 for player A. We can apply dominance and eliminate s_2. That rules out the NE $\{s_2, t_1\}$ which involves a player playing a [weakly] dominated strategy. The solution of this game is therefore $\{s_1, t_2\}$.

7. This type of game is call the "Battle of the Sexes". There are two Nash equilibria $\{s_1, t_2\}$ and $\{s_2, t_1\}$, but neither one is an "obvious" solution to this game.

8. This game is called "Matching Pennies", an example of what is known as zero-sum games because the payoffs associated with each outcome sums to zero (or a constant) . There is no Nash equilibrium in pure strategy, but there is a NE in "mixed" strategy: each player randomizes between their two pure strategies.

9. Another game with no Nash equilibrium in pure strategy.

   - $\{s_1, t_1\}$ is not a NE because player B will want to switch to t_2 instead.
   - $\{s_1, t_2\}$ is not a NE because player A will want to switch to s_2 instead.
   - $\{s_2, t_1\}$ is not a NE because player A will want to switch to s_1 instead.
   - $\{s_2, t_2\}$ is not a NE because player B will want to switch to t_1 instead.

10. This type of game is called a "Game of Chicken". There are two Nash equilibria in this game: $\{s_1, t_2\}$ and $\{s_2, t_1\}$, but neither one is obvious.

1.7 Sequential Move Games

1.7.1 Introduction

There are many situations in which the players take turns to move sequentially

- Game of chess
- The industry leader chooses its output first and the smaller firms pick up the residual demand
- The price leader sets the price and all other firms in the industry follow suit
- One firm launches a marketing campaign and other firms response by launching their own
- An incumbent monopolist makes an investment to deter entry, and a potential entrant decides whether to enter the market or not.

Key Concepts: Threat, Commitments, Credibility, Subgame Perfection

1.7.2 Examples

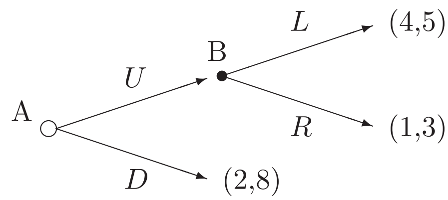
A Simple 2x2 Game

- The players: Player A and Player B

  Strategy Sets: $S_A = \{U, D\}$ and $S_B = \{L, R\}$

  Player A moves first. Player B, after having observed what action Player A has chosen, makes his move. If A chooses D, the game ends. If A chooses U, then it's B's turn to move.

- The extensive form (or game tree) of the game and the payoffs:



$\circ$ = the initial node, $\bullet$ = a decision node

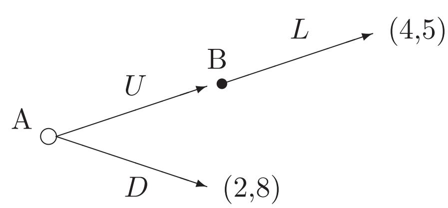
- For Player A, $\{U, L\}$ gives him the highest payoff of 4. So A wants to play U, if he can get Player B to play L. (He can ask himself "What would B do if I played U?")

  For Player B, $\{D, -\}$ gives her the highest payoff of 8. So B wants A to play D and ends the game.

- Since Player B has the last move, can she "threaten" A that she will play R if he plays U so as to force A to play D?

- We will solve this game using **backward induction**: starting at the end of the game with the player who has the last move (Player B) and work our way back to the initial node "o" (the beginning of the game).

- If A plays U and B gets a chance to move, her payoff is 5 if she chooses L and 3 if she chooses R. So we know B will not play R if given the move – her threat is not credible and we can eliminate the lower branch of the game tree labeled R.



- Tracing one step back, A knows if he chooses U Player B would choose L and he will get 4, if he chooses D to end the game he will get 2. So A will choose U.

- The Nash equilibrium of this game is therefore $\{U, L\}$.

- It is easier to see that $\{U, L\}$ is Nash by looking at the strategic (or normal) form of this game

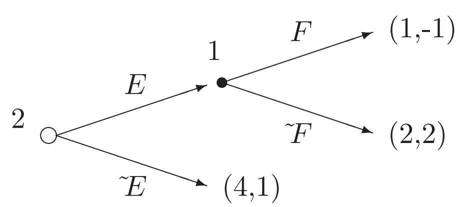
		Player B	
		L	R
Player A	U	$4, 5$	$1, 3$
	D	$2, 8$	$2, 8$

- How many NE are there in this game? There are two: $\{U, L\}$ and $\{D, R\}$.

- Does (weak or strict) dominance apply? Yes, R is weakly dominated by L for Player B so we can eliminate the weak NE $\{D, R\}$ that involves a non-credible strategy.

Entry Deterrence Game

- Two players: Player 1 is an incumbent monopolist and Player 2 is a potential entrant firm.
- Strategy Sets: $S_1 = \{F, \tilde{F}\}$ ($= \{$Fight, Not Fight$\}$) and $S_2 = \{E, \tilde{E}\}$ ($= \{$Enter, Not Enter$\}$)
- The incumbent firm is currently making a profit of 4 (million) and the potential entrant is currently making a profit of 1 (million).
- Player 2 has the first move. If Player 2 chooses $\tilde{E}$, they both continue to make their current profits (4,1).
- If Player 2 decides to enter the monopolist's market, Player 1 can choose to Fight (F) or Not Fight ($\tilde{F}$).
- To successfully fight an entrant will cost the monopolist 3 (million), but the entrant firm will have a net loss of -1 (mil.) because the investment made will be wasted.
- If Player 1 chooses to Not Fight, it will share the market with the new entrant and each makes 2 (mil.)
- This game in extension form:



- The strategic form of this game is

		Player 2: E	Player 2: $\tilde{E}$
Player 1	F	$1, -1$	$4, 1$
	$\tilde{F}$	$2, 2$	$4, 1$

- The incumbent firm can threaten to "fight any entrants at all cost". But is the threat credible?

Now let's find the NE of this game.

- Apply backward induction to the game tree (extensive form).
- Apply weak dominance to the payoff matrix (strategic form).
- Solution: A unique Nash Equilibrium $\{\tilde{F}, E\}$.
- The threat of the incumbent firm to fight is not credible: once the entrant firm enters its market, it is better off not fighting. So the outcome of this game is that the entrant will enter and the incumbent will not fight.

1.8 Subgame Perfection

- A game can have many Nash equilibria and some of them may involve playing strategies (threats or promises) that are not credible – i.e. not in the player's interest to carry them out.
- A threat (to retaliate or punish) is credible only if the player is capable of and is in her interest to carry it out.
- Subgame Perfection is a more "refined" equilibrium concept that can help rule out weaker NE.
- A NE for a game is a **subgame perfect Nash equilibrium** (SGPNE) if it is also a NE for every proper subgame of the game.
- A **proper subgame**: starting at a node of a game tree, if all the nodes and branches from that node on can be treated as a game itself.
- If a game can be solved by backward induction (applicable only to games in extensive form), then the resulting equilibrium is subgame perfect.
- We can also write down the normal form of the game (the payoff matrix) and apply dominance and get the same solution.
- Some forms of commitment (e.g. an incumbent firm making an irreversible investment) which effectively changed the payoffs can make a threat more credible.

1.9 Repeated Play

- Is a cooperative outcome ever possible in a Prisoner's dilemma? Yes, but not if the game is played only once (a one-shot game) or a finite number of times.
- Cooperation can be induced if the game is repeated (i.e., there is repeated interaction between the players) because repetition allows the possibility of retaliation (threats or promises about future actions) and therefore is a mechanism that can help enforce a cooperative agreement.
- Furthermore, reputation matters if there are future interactions.
- A threat (to retaliate or punish) is credible only if the player is capable of and is in her interest to carry it out.
- Strategies to punish your opponent for cheating:
  1. Tit-for-Tat Strategy: begin by cooperating and choose the action that the other player chose in the previous period. That is, if your opponent cooperated in the previous period, you play cooperatively this period; if your opponent cheated in the previous period, punish your opponent by playing non-cooperatively this period but return to cooperate in the following period.
  2. Grim Trigger Strategy: begin by cooperating and continue to play cooperate as long as your opponent cooperates. If your opponent ever defect (cheat), you play non-cooperatively forever after.

- The tit-for-tat strategy is compelling because it is
  - Clear: easy to describe and understand
  - Nice: Start out cooperating
  - Provocable: “one defection and you’ll be punished”
  - Forgiving: punish once and then go back to cooperating

  However, it has a major flaw: one misperception will set off a chain reaction. One side punish the other for (perceived) defecting and the rival hit back with a punishment. This will continue until the next misperception.

- The grim trigger strategy can induce cooperation because the one time gain from cheating will be more than offset by the lower payoffs thereafter from the punishment. But one small misperception can be very costly!

Chapter 2

Exam Questions

Note: The answers are taken directly from their original source with little or no editing.

2.1 May 2005, Q.2

Suppose that it is known in advance that given the number of drivers/cars on the road, there will be 1,000 crashes (one car hitting another) each year in the county of Surrey. Suppose further that the probability of being involved in any such crash is the same for all drivers/cars. However the damage and cost of a crash is different depending on the cars involved. For simplicity, assume that there are two types of cars: Large ones and small ones. Large cars have the advantage that they offer a lot of protection to drivers. They have the disadvantage however that they are expensive to repair.

A crash that involves two small cars typically results in a loss (injury and car damage) of £1,100 for each car. A crash that involves two large cars typically results in a loss of £2,500 for each car. Finally, in a crash that involves one small car and one large car, the driver of the large car suffers damage of £1,800 but the driver of the small car is at such a disadvantage that she actually does even worse and incurs a cost of £3,000. Each driver has to think about what kind of car she wishes to drive recognizing that there is a given chance that she will be in an accident independent of her driving skills.

a. Treating the choice of car as the strategic choice variable, construct the payoff matrix for two drivers depending on the cars that each drives if a crash occurs.

		Driver 2	
		Small Car	Large Car
Driver 1	Small Car	$-£1,100, -£1,100$	$-£3,000, -£1,800$
	Large Car	$-£1,800, -£3,000$	$-£2,500, -£2,500$

b. Does a Dominant Strategies Equilibrium exist in this game? If so, what is it? Explain you answer.

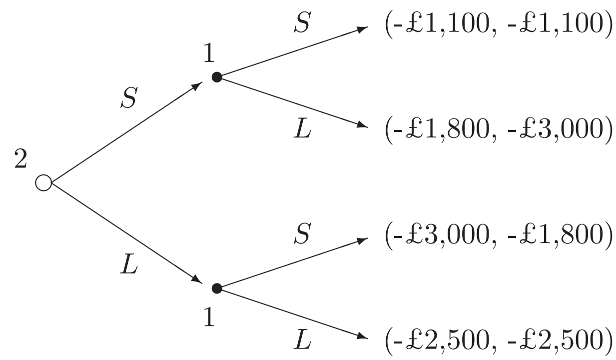
No Dominant Strategies Equilibrium exists for this game since there are no dominated strategies for either players.

c. Assuming the two drivers choose their cars simultaneously, does a Nash Equilibrium exist in this game? If so, what is it? Explain you answer.

There are two Nash Equilibriums to this game {Small, Small} and {Large, Large}. Without a refinement it is not possible to solve this game.

d. What would be the equilibrium if Driver 2 could choose its car first and Driver 1 chose second, taking Driver 2's decisions as given?

First draw the the game tree and then apply backward induction.



The outcome would be that all drivers choose small cars.

e. Based on your analysis, do you foresee any potential inefficiency in the choice of automobiles? Would market forces alone resolve these inefficiencies? Explain.

The population would be better off if all drivers selected small cars only, but left to their own, drivers will choose large cars on the fear that they will suffer more heavily in an accident. This reflects a real world phenomenon where individuals are increasingly purchasing bigger cars for personal safety reasons while increasing the risk to communal safety in doing so. Policies that would eliminate the second Nash Equilibrium (large, large) would, in this case, improve social welfare.

∗ Adapted from D. Richards *Applied Economics for Managers*, 2004

2.2 October 2005, Q.2

Two firms dominate the international market for metal coatings. These are DuPont and BASF. Each has similar costs and production facilities. Each must set its production (measured in millions of tons) several months in advance so that the strategic variable for each is quantity. The BASF profit is given first in each payoff pair.

		DuPont			
		30	40	50	60
BASF	30	\$1700, \$1700	\$1300, \$1900	\$1000, \$2000	\$950, \$1850
	40	\$1900, \$1300	\$1500, \$1500	\$1200, \$1600	\$1100, \$1500
	50	\$2000, \$1000	\$1600, \$1200	\$1300, \$1300	\$1000, \$1200
	60	\$1850, \$950	\$1500, \$1100	\$1200, \$1000	\$900, \$900

a. Discuss the "elements" of a game. Briefly explain how Game Theory can support strategic decisions making. Use examples if you can.

The elements of a game:

1. Players: Make decisions to maximise some objective function. They adopt strategies – a set of rules or game plan that tells them which action to choose at each stage of the game.
2. The payoffs describe what each player receives depending on what strategy was adopted by him and the other players.
3. The equilibrium of a game refers to the combination of strategies in which each of the players in the game has adopted their best strategy. To find the equilibrium of a game, the analyst must specify the players, strategies, payoffs, and equilibrium concept.

Game Theory a strategic tool?

- Game Theory is a set of mathematical techniques used to study interdependent decision-making by agents whose actions affect each other.
- A "game" refers to the conflict of these players
- Players can refer to individuals, firms, political parties, regulatory agencies, governments, etc.
- Game Theory is an extremely popular tool in economics.
- It is based on logic and although mathematical, it provides mainly qualitative insights on strategies.
- John Nash? The famous Nash Equilibrium!
- It is extremely useful and probably the only tool available to study the strategic interactions of firms in Oligopolies.
- Game Theory experts are frequently hired by the private sector to help fine tune their strategic position – big in the USA.

b. In Game Theory, Dominant, Nash, and Subgame Perfect equilibriums are important concepts. Explain what they are and how they differ to each other.

Equilibrium concept:
Defining best strategies is not easy. Ideally, it should:

1. Provide at least one solution to each game (condition of Existence).
2. Should not provide more than one solution (condition of Uniqueness).
3. Solutions should be 'robust' to slight perturbations in the game (condition of stability)
4. Most importantly of all, it should satisfy some sort of common sense criterion.

Examples:

- Dominating Strategy Equilibrium: Based on eliminating what will not happen – recognition that an opponent will not play a dominated strategy defined as one which gives it less whatever his opponent does. Very limited applications!
- Nash Equilibrium: Recognises that the action of one player depends on that of the other player. In equilibrium, no player taking his opponents action as given wishes to change his own actions. Too many solutions!

– Sub-Game Perfectness (Perfect Equilibrium Concept): This is a 'refinement' of the Nash Equilibrium which is based on the notion of eliminating equilibrium that is based on non-credible threats. Add time element.

c. Can you find an equilibrium output for this game assuming that the two firms choose their production quantities simultaneously? Explain.

There is a Dominating Strategy Equilibrium to this game. Firstly, we can eliminate two dominated strategies for each of the players and rearrange the matrix accordingly:

		DuPont Production	
		40	50
BASF Production	40	$1,500, $1,500	$1,200, $1,600
	50	$1,600, $1,200	$1,300, $1,300

Then it becomes apparent that BASF always prefers 50 and so does DuPont. The outcome of the game is (50,50) with a payoff of ($1300, $1300).

∗ The Normal Form was adapted from D. Richards *Applied Economics for Managers*, 2004

2.3 May 2006, Q.3

Two firms (Alpha and Beta) must decide simultaneously whether they should reduce the price of a good they both sell or maintain higher prices (i.e. status quo). The payoffs (Alpha, Beta) they face are as follows:

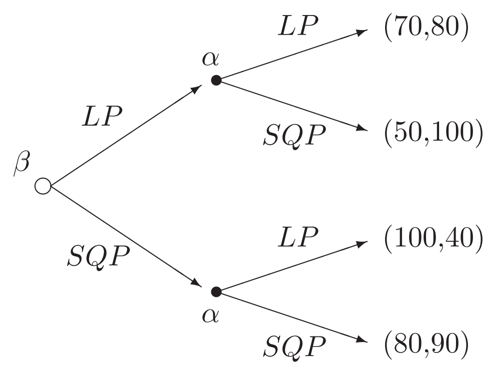
		Beta	
		Lower Price	Status Quo Price
Alpha	Lower Price	70, 80	100, 40
	Status Quo Price	50, 100	80, 90

a. Can you find an 'equilibrium' to this game? Your answer should include a clear definition of the equilibrium concept that you are using and explain coherently what is the solution to this game.

Both firms have a dominating strategy of setting a lower price – i.e. whatever their opponent does it is in their best interest to set a low price. There is therefore a dominating strategy equilibrium in which both firms charge a low price and only make 70 (Alpha) and 80 (Beta) as payoffs. Both would of course prefer the status quo prices but these cannot be maintained when both firms are playing simultaneously.

b. Now suppose that the firms moved in sequence – say firm Beta moved first by setting the low price or the status quo price and firm Alpha could respond to this once she observed what Beta has done. How would you solve such a game? Would the outcome be different? Why or why not? Explain your answer clearly using a diagram.

In this situation you will need to show that the firms move in sequence by drawing a tree diagram showing first Beta with the option of LP or SQP and then Alpha with LP or SQP options at each of the two branches. You should add the respective payoffs to these branch endings. You will need to solve this game backwards by looking at what happens in the last period of the game to find the subgame perfect equilibrium. If you have done this well you will find that the equilibrium strategy of the firms are still charging a low price. Beta cannot in this game induce Alpha to stick to the status quo price and therefore opts to minimise its losses by going for the LP itself. Both would of course prefer the status quo prices but these cannot be maintained even when both firms are playing sequentially.



2.4 October 2006, Q.5

At one time a major US airline proposed that all airlines adopt a uniform fare schedule based on mileage. Doing so would have eliminated the many different fares that were available at the time. Most major airlines applauded the suggestion and began to adopt the plan. Soon however, various airlines began cutting fares. Explain this occurrence using the prisoner's dilemma. (Hint: you don't need to use numbers here!)

These are the assumptions (steps) you needed to take before solving this problem:

- Players: Airline 1 and 2 (you can make a simple argument using only 2 airlines – your conclusions will be applicable to more players)
- Strategies: *Keep High Fares* or *Cut Fares*
- Payoffs (Airline 1, Airline 2): Very low π < Low π < Status Quo π < High π. To figure this out you had to think a bit about the law of demand.

		Airline 2	
		Keep High Fare	Cut Fare
Airline 1	Keep High Fare	status quo π, status quo π	very low π, high π
	Cut Fare	high π, very low π	low π, low π

Airline 1 clearly has a dominating strategy of Cutting Fares, as do Airline 2. By eliminating strategies that will never materialise ("Keeping High Fares"), we find a dominating strategy equilibrium of {Cut Fare, Cut Fare} which explains why airlines cannot keep to a non-enforceable agreement to maintain high fares.

2.5 October 2007, Q.7

Two firms (A and B) are studying their expansion strategies for the coming business cycle. Once they have committed their capital they cannot reverse their decision – at least not in the short term. You can assume that the two firms must decide more or less simultaneously without a knowing what their rivals will actually do and they consider this decision to be a "one-off". If they both expand, prices will fall and they will both end up with 8 millions in revenues each. If they both resist the temptation to expand they can potentially make £9 millions each. However if one of them expands but the other doesn't, then the expanding firm makes £10 millions while the other only makes a miserable £7.5 millions. You can assume that each has similar costs and production facilities.

a). Draw a game theoretical chart or diagram that will synthesise the strategic information contained in the above paragraph. (25 marks)

The table below summarises the payoffs (in millions) associated with each combinations capacity decisions. Firm A's profit is given first in each payoff pair i.e. (£ Firm A, £ Firm B).

		Firm B	
		Not Expand	Expand
Firm A	Not Expand	$9, 9$	$7.5, 10$
	Expand	$10, 7.5$	$8, 8$

b). How would one solve this game? Ensure that you clearly explain how the solution (if one exists) has been obtained and the equilibrium concept(s) tested. (25 marks)

There are many equilibrium concepts but three very popular ones include the following:

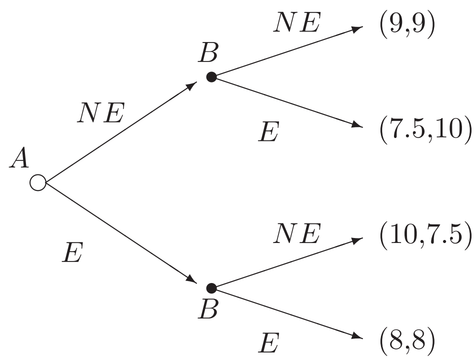
- Dominating Strategy Equilibrium: Based on eliminating what will not happen – recognition that an opponent will not play a dominated strategy defined as one which gives it less whatever his opponent does. Very limited applications!
- Nash Equilibrium: Recognises that the action of one player depends on that of the other player. In equilibrium, no player taking his opponents action as given wishes to change his own actions. Too many solutions!
- Sub-Game Perfectness (Perfect Equilibrium Concept): This is a 'refinement' of the Nash Equilibrium which is based on the notion of eliminating equilibrium that are based on non-credible threats. Add time element.

The first is the easiest and luckily in this case it works well in that it provides a solution to the game. Both firms have a dominant strategy of *Expand* (i.e. they will never NOT expand) which implies that, all else equal, both firms will expand and make 8 millions each.

c). How would your answer change if Firm B moved second after observing Firm A's move? (25 marks)

The solution to the game would be the same. To do so you need to draw a tree diagram with Firm A moving first then solve it backwards as we have done many times in class. You will find that if A exerts restraint and does not expand, Firm B

will have no incentives to restrain itself and capitalise on the opportunity of a higher market share i.e. it will expand. Firm A anticipates this and chooses to expand, and Firm B responds with expansion implying that the equilibrium (*Expand, Expand*) is unaffected by allowing the players to move in sequence.



d). Explain how these answers may be affected if we relaxed the assumption that these firms will 'meet' only once – i.e. would your answers change if the game was not 'one-off'? (25 marks)

Yes – the answers would certainly change and are likely to mimic the results predicted by the folk theorem. What will really matter is if the firms know when the last period will be. If this is uncertain then the firms will tend to cooperate more and the (*Not Expand, Not Expand*) outcome becomes a more feasible one. The most aggressive and consumer friendly results are often obtained in short-run one-off games between rival firms in this sort of context.

Chapter 3

The Dynamics of Pricing Rivalry

This chapter is not important for the exam, but it will give you a better understanding of the nature of competition in a dynamic (vs. static) setting.

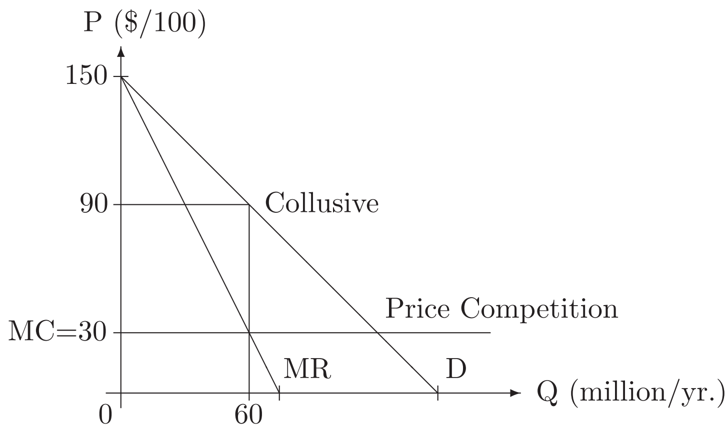
3.1 Why the Cournot and Bertrand models are not dynamic?

- Recall the pseudo-dynamic story we told in the Oligopoly chapter: a firm chooses its quantity or price based on what its rival did in its previous move $\Rightarrow$ its response is the choice that maximizes its current profit.
- But an intelligent firm would take the long view and choose its quantity or price to maximize long-term profit – the present value of profits over its entire time horizon.
- To do this, a firm must anticipate what its rival will do in the future, not just naívely react to what it has done in the past.
- Cournot and Bertrand models are not dynamic because:
  - there is no real convergence process
  - there are no time elements
- Adding dynamic considerations is important because they may
  - explain why oligopoly prices are not always driven to marginal cost
  - generate ideas of *tacit* rather than *explicit* collusion.

3.2 Dynamic Pricing Rivalry: Intuition

- Cournot and Bertrand competitors
  - prefer prices that are close to the monopoly level.
  - prefer to avoid price competition.
- Consider the following example (see diagram below):
- Collusion will give each firm an annual profit of \$18 million $= ((90 - 30) \times 60)/2$.

  Competition will drive price down to MC = 30 and profit to zero.



- In a two-firm market, the two competitors could collude and charge the monopoly price, but it is illegal in most countries.
- The questions we are interested in are
  - Can competitors find a way to **tacitly cooperate**?
  - Are there conditions under which a firm might not wish to undercut its rivals (either by lowering its price relative to theirs or refusing to go along with their price increase)?
- A firm contemplates undercutting its rivals faces a tradeoff.
  - It stands to reap a short-run increase in profits, or a long-run increase if the price reduction translates into an increase in market share.
  - The firm's rivals might respond by lowering their own prices. Once they do, the firm that initiated the price reduction could end up with no increase in market share, but with lower price-cost margins.
- Aggression today (undercutting) may not pay in the future!

3.2.1 Competitor Responses and Tit-for-Tat Pricing

- Suppose the two firms are currently charging $P = \$70$ (Cournot price).
- If firm A decides to raise its price to \$90, it's profit will drop to zero unless firm B decides to raise its price to \$90 as well.
- Will / should firm B follow suit?

  If firm B follows the price increase, it will have a profit increase of \$2 million per year. (Verify this as an exercise.) The discounted present value is

$$\pi_f = 2 + \frac{2}{(1+i)} + \frac{2}{(1+i)^2} + \frac{2}{(1+i)^3} + \dots$$

  If firm B ignores the price increase it will have a one-off profit increase of \$16 million.

$$\pi_i = \pi_c = 16$$

- Ignore the price increase is a better option for firm B if

$$\begin{aligned}
\pi_i &> \pi_f \\
16 &> 2 + \frac{2}{(1+i)} + \frac{2}{(1+i)^2} + \frac{2}{(1+i)^3} + \cdots \\
14 &> \frac{2}{(1+i)} + \frac{2}{(1+i)^2} + \frac{2}{(1+i)^3} + \cdots \\
14 &> 2/i \\
i &> 1/7 = 14.3\%
\end{aligned}$$

  Firm B is better off ignoring the price increase if the discount rate is 14.3% per annum.

- What if firm A can increase its price but reverse it in one week? If firm B does not follow suit, firm A loses at most one week's profit. Let $r = i/52$ be the weekly discount rate.

  The discounted present value of additional profit to firm B if it follows the price increase

$$\pi_f = 2/52 + \frac{2/52}{(1+r)} + \frac{2/52}{(1+r)^2} + \frac{2/52}{(1+r)^3} + \cdots$$

  Firm B's additional profit from ignoring the price increase

$$\pi_i = \pi_c = 16/52$$

- Ignore the price increase is a better option for firm B if

$$\begin{aligned}
\pi_i &> \pi_f \\
\frac{16}{52} &> 2/52 + \frac{2/52}{(1+r)} + \frac{2/52}{(1+r)^2} + \frac{2/52}{(1+r)^3} + \cdots \\
14 &> \frac{2}{(1+r)} + \frac{2}{(1+r)^2} + \frac{2}{(1+r)^3} + \cdots \\
14 &> 2/r \\
r &> 1/7 = 14.3\% \quad \text{per week!}
\end{aligned}$$

** Conclusion: The faster a price change can be reversed, the more likely a tacit coordination is sustainable

- The outcome corresponds to the monopoly outcome even though the firms did not collude with each other! (That's why we call it "tacit coordination".)

Tit-for-Tat

- Tit-for-Tat: "I will charge in the current period whatever price you charged in the previous period."

- If the two firms can commit to playing the tit-for-tat strategy, they may be able to avoid costly price wars.

- How to commit to this strategy? Without commitment the strategy has no "bite".

** The tit-for-tat strategy is akin to a commitment by firms to its customers that "We will NOT be undersold" or "Lowest Price Guarantee – we'll refund the difference" because the customers inadvertently become enforcing agents who help detect cheating, hence provide strong incentive for firms to NOT deviate their prices from their rivals.

- The tit-for-tat strategy not only discourages firms from cutting price to steal business from competitors, it can also encourage firms to raise prices toward the monopoly level!

3.3 Facilitating Practices

Firms can adopt practices that can either facilitate coordination (tacit cooperation) among firms or diminish their incentive to cut price.

- Price leadership
- Advanced announcement of price changes
- Most favored customer clauses
- Uniform delivered pricing

3.3.1 Price Leadership

- One firm in an industry takes the lead in price change and others follow the leader's price. Examples: Kellogg (cereal), Saudi Arabia (oil).
- Firms cede control over industry pricing to a single firm to facilitate coordination. But it can raise anti-trust concerns because price matching is a sign of collusion.
- Oligopolistic price leadership vs. barometric price leadership

  Under barometric price leadership, the price leader merely acts as a barometer of changes in market conditions by adjusting prices to shifts in demand or input prices. So different firms may act as price leaders at different times. Example: airlines

  Under oligopolistic leadership, the same dominant firm is usually the leader.

3.3.2 Advance Announcement of Price Changes

- Firms will publicly announce the prices they intend to charge in the future.
- The announcement allows rivals time to match and the announcer a chance to rescind if the rivals don't follow.

3.3.3 Most Favored Customer Clauses

- A most favored customer clause is a provision in a sales contract that promises a buyer that she will pay the lowest price the seller charges.
- Two basic types: contemporaneous and retroactive.
  1. Contemporaneous: while the contract is in effect, any lower price charged to another customer will generate a rebate.

2. Retroactive: a rebate will be given over some period of time from the signing date of the contract if a lower price is charge to another customer.

- Most favored customer clauses appear to benefit buyers, but it actually helps soften price competition:
  - Retroactive most favored customer clauses make it expensive for a seller to cut prices in the future, either selectively or across the board.
  - Contemporaneous most favored customer clauses do not penalize a seller for making across-the-board price reductions, but they discourage a seller from using selective price cutting to compete for customers with are price elastic demands.

3.3.4 Uniform Delivered Prices

In many industries, buyers and sellers are geographically separated, and transportation costs are can be significant.

- FOB (free on board) pricing

  The seller quotes a price for pickup at the seller's loading dock, and the buyer absorbs the freight and delivery charges.

  If one firm lowers its FOB prices, price matching is expensive because its rivals have to lower their prices to ALL locations.

- Uniform Delivery Pricing

  The seller quotes a single delivered price for all buyers and absorbs any freight charges itself.

  If one firm reduces its [uniform] delivery price to one particular location, its rivals could cut its price selectively.

  It facilitates cooperative pricing by allowing sellers to make a more surgical response to price cutting by rivals and keep delivered prices of customers at other locations the same.

  Uniform delivery pricing makes price matching less expensive and retaliation more likely (playing the tit-for-tat strategy) $\Rightarrow$ helps sustain cooperative pricing.

1.

		B	
		t_1	t_2
A	s_1	$6, 6$	$2, 3$
	s_2	$3, 2$	$1, 1$

2.

		B	
		t_1	t_2
A	s_1	$2, 7$	$0, 4$
	s_2	$5, 1$	$-4, -2$

3.

		B	
		t_1	t_2
A	s_1	$2, 7$	$0, 4$
	s_2	$5, 1$	$-4, 1$

4.

		B	
		t_1	t_2
A	s_1	$5, 5$	$-3, 8$
	s_2	$8, -3$	$0, 0$

5.

		B	
		t_1	t_2
A	s_1	$10, 10$	$0, 0$
	s_2	$0, 0$	$1, 1$

6.

		B	
		t_1	t_2
A	s_1	$10, 0$	$5, 2$
	s_2	$10, 1$	$2, 0$

7.

		B	
		t_1	t_2
A	s_1	$0, 0$	$81, 80$
	s_2	$80, 81$	$0, 0$

8.

		B	
		t_1	t_2
A	s_1	$1, -1$	$-1, 1$
	s_2	$-1, 1$	$1, -1$

9.

		B	
		t_1	t_2
A	s_1	$3, 1$	$1, 3$
	s_2	$0, 5$	$4, 2$

10.

		B	
		t_1	t_2
A	s_1	$-3, -3$	$2, 0$
	s_2	$0, 2$	$1, 1$

Solve the following games using iterative dominance:

		Player B		
		L	C	R
Player A	U	$1, 0$	$1, 2$	$0, 1$
	D	$0, 3$	$0, 1$	$2, 0$

NE: $\{U, C\}$

(R, D, L)

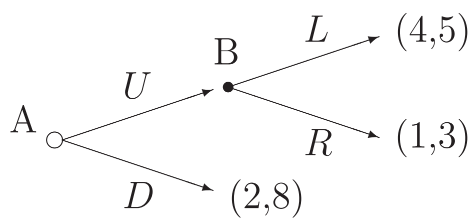
		Player B		
		L	C	R
	T	$2, 0$	$1, 1$	$4, 2$
Player A	M	$3, 4$	$1, 2$	$2, 3$
	B	$1, 3$	$0, 2$	$3, 0$

NE: $\{M, L\}$ and $\{T, R\}$

(B, C)

A Simply Sequential Move Game

The game tree:

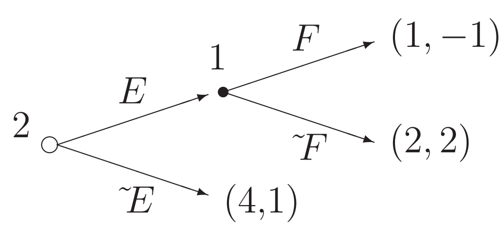


The strategic form of this game:

		Player B	
		L	R
Player A	U	$4, 5$	$1, 3$
	D	$2, 8$	$2, 8$

An Entry Deterrence Game

The game tree:



The strategic form of this game:

		Player 2	
		E	˜E
Player 1	F	$1, -1$	$4, 1$
	˜F	$2, 2$	$4, 1$